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THE SUMMER MEETING IN MINNEAPOLIS

The thirty-seventh summer meeting and fifteenth colloquium of the Society were held at the University of Minnesota, Minneapolis, Minnesota, from Tuesday to Friday, September 8–11, 1931. The meeting was preceded by that of the Mathematical Association of America. About two hundred forty persons attended the meeting, among whom were the following one hundred forty-six members of the Society:

C. R. Adams, R. B. Adams, N. L. Anderson, R. C. Archibald, W. L. Ayres, R. H. Bardell, R. W. Barnard, Herman Betz, W. J. Berry, G. A. Bliss, M. G. Boyce, C. F. Bowles, W. C. Brenke, R. W. Brink, W. E. Brooke, H. E. Buchanan, J. A. Bullard, W. H. Bussey, W. D. Cairns, J. W. Campbell, Elizabeth Carlson, W. B. Carver, W. J. Cheney, E. W. Chittenden, G. R. Clements, A. B. Coble, C. J. Coe, Abraham Cohen, Julia Colpitts, R. P. Conkling, L. P. Copeland, A. T. Craig, C. F. Craig, E. H. Cutler, H. H. Dalaker, M. E. Daniels, L. L. Dines, J. M. Earl, M. C. Eide, G. C. Evans, H. P. Evans, H. S. Everett, K. W. Folley, W. B. Ford, Philip Franklin, T. C. Fry, H. J. Gay, J. J. Gergen, Gladys Gibbens, C. Gouwens, L. M. Graves, L. W. Griffiths, L. D. Hampton, W. L. Hart, M. L. Hartung, E. R. Hedrick, M. R. Hestenes, E. H. Hildebrandt, Einar Hille, J. J. L. Hinrichsen, P. G. Hoel, D. L. Holl, H. M. Hosford, E. V. Huntington, J. W. Hurst, Louis Ingold, M. H. Ingraham, Dunham Jackson, C. M. Jensen, B. W. Jones, W. H. Kirchner, E. P. Lane, H. I. Lane, R. E. Langer, C. G. Latimer, Lincoln La Paz, A. D. Lewis, C. I. Lubin, N. H. McCoy, C. C. MacDuffee, W. H. McEwen, J. V. McKelvey, M. M. McKelvey, N. B. MacLean, A. I. McMaster, W. D. MacMillan, A. J. Maria, H. W. March, Morris Marden, E. W. Miller, I. L. Miller, Norman Miller, W. L. Miser, C. N. Moore, W. L. Moore, Elmer Moots, D. C. Morrow, D. S. Morse, Marston Morse, T. A. Mossman, C. O. Oakley, Louis O'Shaughnessy, F. W. Owens, H. B. Owens, Gordon Pall, H. P. Pettit, T. Rado, Ruth Rasmussen, S. E. Rasor, W. T. Reid, B. L. Remick, R. G. D. Richardson, Paul Rider, H. L. Rietz, W. C. Risselman, P. G. Robinson, W. H. Roever, A. E. Ross, W. E. Roth, R. J. Sanger, I. J. Schoenberg, E. W. Sheldon, J. W. Shuman, R. R. Shumway, L. P. Siceloff, E. B. Skinner, H. E. Slaught, C. E. Smith, G. W. Smith, H. F. Smith, Virgil Snyder, E. B. Stouffer, Eugene Taylor, H. L. Turrittin, J. V. Uspensky, H. S. Vandiver, L. E. Ward, C. W. Watkeys, Warren Weaver, W. P. Webber, M. J. Weiss, Marian Wilder, E. W. Wilson, Marian White, Fredrick Wood, J. W. Young.

The colloquium lectures on the *Calculus of Variations* by Professor Marston Morse were delivered on Tuesday afternoon and Wednesday, Thursday and Friday mornings. A summary of these lectures will appear in a later issue of this Bulletin. By invitation of the Committee on Program, Professors Edmund Landau and C. C. MacDuffee gave, on Wednesday and Thurs-

day mornings respectively, addresses entitled *Schnirelmann's theorem* and *Ideals in linear algebras*.

On Monday evening the University of Minnesota tendered a reception to the visiting mathematicians. This was followed by cards, dancing and other entertainment. On Tuesday noon there was a luncheon for ladies at the Lafayette Club on Lake Minnetonka. On Wednesday evening the joint dinner of the Society and the Mathematical Association of America was held at the Minneapolis Automobile Club. Professor Dunham Jackson acted as toastmaster and called upon President Coffman of the University of Minnesota who welcomed the visitors to the University, Professor E. R. Hedrick who spoke of the plans for the meeting in California next summer, Miss Marie Weiss who urged upon the members the desirability of attendance at the New Orleans meetings and spoke of the many interesting features of New Orleans. Professor N. B. MacLean of McGill University spoke of the pleasant relationship existing between Canadian mathematicians and those of the United States, and Professor Warren Weaver gave the concluding talk of the evening. Thursday afternoon was devoted to an excursion to the Chisago Lakes and Taylor's Falls.

After lunch on Wednesday a group photograph of those in attendance was taken.

The Minneapolis Art Institute was open without fee to members of the Society. The golf course and tennis courts of the University of Minnesota were available to those in attendance at the meeting.

Most of the visitors were housed in Sanford Hall, and meals were served in its dining room. This building also served as a social center for the meeting. The arrangements which Professor Brink and the various local committees had made for the entertainment and comfort of the members of the Society and their friends were most successfully planned and carried out. All those present felt deeply grateful to the Minnesota group for their generous and thoughtful hospitality. This feeling was voiced by Professor Virgil Snyder at the general meetings on Thursday morning at which a rising vote of thanks was given to the local committees and others that had aided in preparing and carrying out the plans for the meeting.

The Council met on the evening of September 8 and various actions taken at this time were reported to the Society.

Yale University was elected to a Sustaining Membership.

The following seventy-three persons were elected to membership in the Society:

Dr. Dennis Burley Ames, Yale University;
Mr. Horace Crookham Ayres, University of Washington;
Mr. Aaron Bakst, Columbia University;
Dr. Arthur F. Bentley, Paoli, Indiana;
Dr. Henri Frédéric Bohnenblust, Princeton University;
Dr. Leonard Bristow, University of Illinois;
Mr. Donald Meeker Brown, University of Illinois;
Mr. John A. Carlson, University of Washington;
Mr. Philip Nathaniel Carpenter, University of Washington;
Miss Mary Frances Cleary, Boston;
Mr. George Comenetz, Columbia University;
Mr. Edwin G. H. Comfort, Wilmette, Ill.;
Miss Helen Marian Copenhagen, University of Washington;
Mr. Samuel Arthur Corey, Des Moines;
Dr. Allen Thornton Craig, University of Iowa;
Mr. Walter Sindall Dawkins, Baltimore;
Mr. Dewey S. Dearman, University of Cincinnati;
Mr. Richard Bolling Dunn, Yale University;
Mr. Milo Alembert Durand, Columbia University;
Mr. John K. Dyer, Yale University;
Mr. Ralph Asbury Edmondson, Wiley College;
Mr. George McNaught Ewing, University of Missouri;
Professor Donald Faulkner, Stetson University;
Professor Elmer Colin Goldsworthy, University of California;
Miss Mary Elizabeth Haller, University of Washington;
Mr. Charles Henry Harry, The Johns Hopkins University;
Professor Frances Harshbarger, Constantinople Woman's College;
Professor Thomas Watkins Hatcher, Virginia Polytechnic Institute;
Professor Leland David Hemenway, Simmons College;
Mr. Paul Herget, University of Cincinnati;
Professor Harry Grant Hodgkins, The George Washington University;
Miss Grace Murray Hopper, New York City;
Professor Howard Kibble Hughes, Purdue University;
Mr. Evan Johnson, Jr., Pennsylvania State College;
Miss Helen M. Kammerer, American Telephone and Telegraph Company;
Mr. Arthur James Kavanagh, Gunnery School, Washington, Conn.;
Mr. John Francis Kenney, Northwestern University;
Dr. Voris V. Latshaw, Fort Hays Kansas State College;
Mr. Reuben McDaniel, Shaw University;
Dr. Henry Jarvis Miles, University of Illinois;
Miss Alta Odoms, University of Cincinnati;
Miss Gladys Martha Quigg, Pennsylvania State College;

Mr. Lewis M. Reagan, Brooklyn Polytechnic Institute;
Mr. John Riordan, American Telephone and Telegraph Company;
Mr. Benjamin Rosenbaum, Milford School, Milford, Conn.;
Mr. Albert Louis Ruiz, General Electric Company, Schenectady;
Dr. William Monroe Rust, Jr., Rice Institute;
Dr. Isaac J. Schoenberg, University of Jassy, Jassy, Roumania, International Research Fellow;
Mr. Ellsworth Elmer Strock, Cleveland Heights, Ohio;
Mr. Erling D. Sunde, American Telephone and Telegraph Company;
Director William Francis Gray Swann, Bartol Research Foundation of the Franklin Institute, Swarthmore, Pa.;
Mr. Joseph Fraser Thomson, University of Michigan;
Mr. Hsien San-Tsang, Ithaca, N. Y.;
Professor Ralph Sylvester Underwood, Texas Technological College;
Dr. Samuel Stanley Wilks, Columbia University;
Dr. Aurel Frederick Wintner, The Johns Hopkins University;
Dr. Henry Marcus Yarbrough, Western Kentucky State Teachers College;
Dr. Robert Carl Yates, The Johns Hopkins University;
Miss Marguerite Zeigel, University of Missouri;

The Secretary announced the election of the following persons to membership by mail vote of the Council:

Professor Harald Bohr, University of Copenhagen;
Professor Carl Syfan Cox, Southern College;
Mr. Joseph Leo Doob, Harvard University;
Mr. Marshall Hall, Yale University;
Mr. Sumner Byron Myers, Harvard University;
Mrs. Dorothy Pennock Nile, Columbia University;
Professor Brendon O'Connor, Catholic University, Peking;
Professor Maurice Marie Jean Potron, Ecole Sainte Geneviève, Paris;
Professor Wallace F. Smith, New River State College, Montgomery, W. Va.;

As nominees of Members of the Departments of Mathematics of Lehigh and George Washington Universities:

Mr. Edwin L. Bartleson, U. S. Weather Bureau;
Mr. Abraham Sinkov, U. S. Civil Service;
Mr. Welby Reed Stevens, U. S. Weather Bureau;
Mr. Henry George Swain, Lehigh University;
Mr. Albert Wertheimer, Bureau of Ordnance, U. S. Navy.

The Joint Committee on Funds has been enlarged and now consists of Professors R. C. Archibald, G. D. Birkhoff, G. A. Bliss, W. D. Cairns, A. B. Coble, J. L. Coolidge, Tomlinson Fort, Drs. T. C. Fry, J. W. Glover, Professors E. R. Hedrick, M. H. Ingraham, G. W. Mullins, H. E. Slaught, D. E. Smith and J. W. Young (chairman).

Invitations for the winter meeting of 1932 were received from

Bryn Mawr College, Duke University, and the George Washington University.

The Summer Meeting of 1932 in Los Angeles was set for the period August 30—September 2 and the following persons were appointed a Committee on Arrangements: Professor E. R. Hedrick, Chairman, Professors E. T. Bell, M. H. Ingraham, T. M. Putnam, and W. M. Whyburn.

The only other meeting on the Pacific Coast for 1932 is to be at Stanford University on April 9.

The following persons were reported as appointed to represent the Society: at the inauguration of Robert H. Ruff as President of Central College, Fayette, Mo., May 25, 1931, Professor W. D. A. Westfall; at the meeting of the Affiliation Committee of the American Association for the Advancement of Science, Pasadena, June 16, 1931, Professors E. T. Bell and E. R. Hedrick; at the dedication of the Lakeside Hospital, Cleveland, Ohio, June 17, 1931, Professor F. C. Jonah; at the Faraday Celebrations, sponsored by the Royal Institution of Great Britain, held in London, England, September 21–25, 1931, Professor G. H. Hardy; at the Seventh American Scientific Congress sponsored by the Pan American Union and to be held in Mexico City during February 1932, Professor Oswald Veblen.

In connection with the World's Fair to be held in Chicago in 1933 there is to be an important exhibition showing the progress of science during the past century. This is to include a mathematics exhibit and it is planned that the Society hold its Summer Meeting in Chicago at the time of the meeting of the American Association for the Advancement of Science probably during the last week of June. The Fair is inviting a group of foreign scientists to lecture and a committee of the Society has made recommendations regarding personnel in mathematics.

The titles and cross references to abstracts of the papers read at the sessions follow below. The papers whose abstract numbers are followed by the letter *t* were read by title, the rest in person. The papers were read as follows: papers numbered 1 to 5 before a general session on Tuesday afternoon at which Professor E. R. Hedrick presided; papers numbered 6 to 23, 53, 71, 73, and 74 Wednesday afternoon in a section on Analysis, Professor Warren Weaver presiding; papers numbered 24 to 50

and 72 Wednesday afternoon in a section on Algebra and Topology, Professors C. G. Latimer and C. C. MacDuffee presiding; and papers numbered 51, 53, 54 to 69, and 75 at a general session on Friday morning with Professor Virgil Snyder presiding. Professor Franklin presented paper No. 6, Professor W. B. Ford paper No. 7, and Dr. Hartung paper No. 11. Professor Hassel was introduced by Professor Albert, Dr. Hopf by Professor Tamarkin, Mr. Scherberg and Dr. Edgett by Professor Ingraham, and Dr. Robinson by Professor E. W. Chittenden.

1. *On linear inequalities*, by Professor L. L. Dines and Dr. N. H. McCoy (National Research Fellow). (Abstract No. 37–9–278.)

2. *Transformations of double sequences, with application to Cesàro summability of double series*, by Professor C. R. Adams. (Abstract No. 37–9–279.)

3. *Note on the application of Markoff's theorem to problems of approximation in the complex domain*, by Professor Dunham Jackson. (Abstract No. 37–5–233.)

4. *On Jordan continua that are the common boundaries of two or more domains in* E_n, by Professor R. L. Wilder. (Abstract No. 37–7–258–*t*.)

5. *The Cauchy integral and summable boundary functions*, by Professor G. C. Evans. (Abstract No. 37–9–280.)

6. *Dual Pfaffians*, by Professors C. L. E. Moore and Philip Franklin. (Abstract No. 37–9–281.)

7. *On the behavior of entire functions in distant portions of the plane*, by Professor C. V. Newsom. (Abstract No. 37–9–282.)

8. *On sets of constants which are not the Fourier constants of a function integrable* (*L*), by Professor C. N. Moore. (Abstract No. 37–9–283.)

9. *On Stieltjes polynomials* (second paper), by Professor Morris Marden. (Abstract No. 37–7–271.)

10. *On the asymptotic forms of the solutions of differential equations*, by Professor R. E. Langer. (Abstract No. 37–9–284.)

11. *On a class of integral equations with a finite discontinuity in the kernel*, by Professor R. E. Langer and Dr. M. L. Hartung. (Abstract No. 37–9–285.)

12. *Some extensions of the Schwarz-Stieltjes theorem of mean value and some consequences deduced therefrom*, by Professor Norman Miller. (Abstract No. 37–9–286.)

13. *On the condition of Weierstrass in the problem of Lagrange*, by Professor L. M. Graves. (Abstract No. 37–9–287.)

14. *A boundary value problem associated with the calculus of variations*, by Dr. W. T. Reid (National Research Fellow). (Abstract No. 37–9–288.)

15. *Note on multiple Dirichlet and multiple factorial series*, by Professor C. R. Adams. (Abstract No. 37–9–289–*t*.)

16. *Note on Volterra and Fredholm products of symmetric kernels*, by Dr. L. M. Blumenthal. (Abstract No. 37–7–262–*t*.)

17. *An inequality for the Bessel functions of the first kind with imaginary argument*, by Dr. T. H. Gronwall. (Abstract No. 37–7–276–*t*.)

18. *On Stieltjes continued fractions*, by Professor J. A. Shohat. (Abstract No. 37–9–290–*t*.)

19. *On some applications of Parseval's formula*, by Professor J. A. Shohat. (Abstract No. 37–9–291–*t*.)

20. *The k-function, a particular case of the confluent hypergeometric function*, by Professor Harry Bateman. (Abstract No. 37–9–292–*t*.)

21. *On a theorem of S. Bernstein-Widder*, by Professor J. D. Tamarkin. (Abstract No. 37–9–293–*t*.)

22. *Complexes and dimension theory*, by Professor G. C. Evans. (Abstract No. 37–9–294–*t*.)

23. *The Euler-Lagrange multiplier rule for double integrals*, by Mr. Max Coral. (Abstract No. 37–7–273–*t*.)

24. *On the equation* $P(A, X)=0$ *in matrices*, by Professor W. E. Roth. (Abstract No. 37–9–295.)

25. *A correspondence between classes of matrices and classes of ideals*, by Professor C. G. Latimer. (Abstract No. 37–9–296.)

26. *On the method of infinite descent and Fermat's last theorem for regular prime exponents*, by Professor H. S. Vandiver. (Abstract No. 37–9–297.)

27. *A supplement to the law of inertia for real quadratic forms with an application to the projective geometry of hyperquadrics*, by Dr. I. J. Schoenberg (International Research Fellow). (Abstract No. 37–7–274.)

28. *On Selling's reduction of positive ternary quadratic forms*, by Professor B. W. Jones. (Abstract No. 37–7–266.)

29. *On the resultant of a system of forms homogeneous in each*

of several sets of variables, by Dr. N. H. McCoy (National Research Fellow). (Abstract No. 37–7–260.)

30. *Note on a property of continuous arcs*, by Professor W. L. Ayres. (Abstract No. 37–9–298.)

31. *On a kind of homogeneity in continuous curves*, by Dr. E. W. Miller. (Abstract No. 37–9–299.)

32. *An application of metric geometry to determinants*, by Dr. L. M. Blumenthal. (Abstract No. 37–7–263–*t*.)

33. *A note on cyclic algebras of order sixteen*, by Professor A. A. Albert. (Abstract No. 37–7–267–*t*.)

34. *On the construction of cyclic algebras with a given exponent*, by Professor A. A. Albert. (Abstract No. 37–7–268–*t*.)

35. *Division algebras over an algebraic field*, by Professor A. A. Albert. (Abstract No. 37–7–269–*t*.)

36. *Algebras without moduli which are equivalent to the algebras of their first and their second matrices*, by Mr. L. E. Bush. (Abstract No. 37–9–300–*t*.)

37. *Theory of cyclic algebras over an algebraic field*, by Professor H. Hasse. (Abstract No. 37–9–301–*t*.)

38. *On rational transformations*, by Professor Raymond Garver. (Abstract No. 37–9–302–*t*.)

39. *On the second factor of the class number of a cyclotomic field*, by Professor H. S. Vandiver. (Abstract No. 37–9–303–*t*.)

40. *On the norm-residue symbol in the theory of cyclotomic fields*, by Professor H. S. Vandiver. (Abstract No. 37–9–304–*t*.)

41. *Summary of results and proofs on Fermat's last theorem* (sixth paper), by Professor H. S. Vandiver. (Abstract No. 37–9–305–*t*.)

42. *Two theorems concerning abstract spaces*, by Professor R. G. Lubben. (Abstract No. 37–7–264–*t*.)

43. *Concerning regular accessibility*, by Miss Beatrice Aitchison. (Abstract No. 37–7–265–*t*.)

44. *An examination of some cut sets of space*, by Mr. C. H. Harry. (Abstract No. 37–7–270–*t*.)

45. *Concerning non-locally compact spaces*, by Mr. C. H. Harry. (Abstract No. 37–9–306–*t*.)

46. *On cyclic numbers of one-dimensional compact sets*, by Professor W. A. Wilson. (Abstract No. 37–9–307–*t*.)

47. *On upper semi-continuous decompositions of compact continua*, by Professor W. A. Wilson. (Abstract No. 37–9–308–*t*.)

48. *A junction property of locally connected sets*, by Professor G. T. Whyburn. (Abstract No. 37–9–309–*t*.)

49. *Regular chains and their application to proofs in point set theory*, by Professor G. T. Whyburn. (Abstract No. 37–9–310–*t*.)

50. *Group invariants and torsion coefficients*, by Dr. A. B. Brown. (Abstract No. 37–7–259–*t*.)

51. *Examples of harmonic functions*, by Professor A. J. Maria. (Abstract No. 37–9–311.)

52. *On deferred Cesàro means*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 37–7–261–*t*.)

53. *The degree of convergence of a series of Bessel functions*, by Mr. M. G. Scherberg. (Abstract No. 37–7–272–*t*.)

54. *An invariantive classification of plane cubic curves under the affine group*, by Dr. R. S. Burington. (Abstract No. 37–9–312–*t*.)

55. *On a covariant differentiation process*, by Professor H. V. Craig. (Abstract No. 37–9–313–*t*.)

56. *Characteristic properties of the euclidean length integral*, by Professor Lincoln LaPaz. (Abstract No. 37–9–314–*t*.)

57. *On a method of summation of infinite series and geometrical interpretation of the method*, by Mr. E. A. Kholodovsky. (Abstract No. 37–7–257–*t*.)

58. *Vector treatment of the displacements of a rigid body*, by Professor C. J. Coe. (Abstract No. 37–9–315.)

59. *A simplified theory of the figure of the earth* (preliminary communication), by Professor E. V. Huntington. (Abstract No. 37–9–316.)

60. *Systems of polynomials connected with the Pearson differential and difference equations*, by Professor E. H. Hildebrandt. (Abstract No. 37–9–317.)

61. *On the distribution of the correlation coefficient in small samples*, by Professor P. R. Rider. (Abstract No. 37–9–318.)

62. *The probability limit theorem*, by Professor A. H. Copeland. (Abstract No. 37–9–319–*t*.)

63. *Frequency distributions with given statistics which are not all moments*, by Dr. G. L. Edgett. (Abstract No. 37–9–320–*t*.)

64. *On the Tchebycheff inequality of Bernstein*, by Dr. C. C. Craig (National Research Fellow). (Abstract No. 37–9–321–*t*.)

65. *On the theory of potentiometric titration*, by Dr. T. H. Gronwall. (Abstract No. 37–7–275–*t*.)

66. *A special conformal space of three dimensions occurring in wave mechanics*, by Dr. T. H. Gronwall. (Abstract No. 37–7–277–*t*.)

67. *Two notes on the mathematics of finance*, by Professor Raymond Garver. (Abstract No. 37–9–322–*t*.)

68. *Circular trajectories in dynamics*, by Professor Edward Kasner. (Abstract No. 37–7–253–*t*.)

69. *Proof of a generalized form of Newton's law of gravity*, by Professor O. E. Glenn. (Abstract No. 37–7–255–*t*.)

70. *Arithmetical composition and inversion of functions over classes*, by Professor E. T. Bell. (Abstract No. 37–9–323–*t*.)

71. *On the uniqueness of the solution of the problem of Plateau*, by Professor Tibor Rado. (Abstract No. 37–9–324.)

72. *Concerning S-regions in locally connected continua*, by Professor G. T. Whyburn. (Abstract No. 37–9–325–*t*.)

73. *Theory of measure and invariant integrals*, by Dr. Eberhard Hopf. (Abstract No. 37–9–326–*t*.)

74. *Poisson's equation with relation to sub-harmonic functions*, by Professor G. C. Evans. (Abstract No. 39–9–327–*t*.)

75. *Concerning theorems in general topology*, by Dr. Selby Robinson. (Abstract No. 39–9–328.)

MARK H. INGRAHAM,
Associate Secretary

THE PASADENA MEETING OF SECTION A AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

Section A (Mathematics) of the American Association for the Advancement of Science held two sessions for the reading of three invited papers on Tuesday, June 16, 1931, during the week of the Spring Meeting of the Association at Pasadena, California. Attendance at the sessions averaged about fifty. The chairman, Professor E. R. Hedrick, prefaced the reading of the papers in the morning session by calling the attention of those present to a tentative list of foreign mathematicians to be invited to give lectures at the Chicago 1933 Congress, in connection with the meeting of the Association and the American Mathematical Society.

Professor D. R. Curtiss of Northwestern University, presented an interesting paper entitled *Certain diophantine problems of approximation*. Professor Curtiss gave an indication of various diophantine problems treated by him in a paper published in this Bulletin in 1929. He pointed out some applications connected with the conformal mapping of a plane on a triangle and the covering of a plane with regular polygons and then went on to illustrate the method of exhaustions. Professor Curtiss commented on some unsolved problems and in particular on an asymptotic formula for the number of integer solutions of an algebraic equation with integer coefficients subject to auxiliary conditions.

Professor O. D. Kellogg, of Harvard University, presented the second paper of the morning session, on *Topics in potential theory*. He prefaced the main part of his talk with a historical introduction to the Dirichlet problem. He pointed out that it was not until 1913 that Zaremba showed that the Dirichlet problem with continuous boundary values is not possible for every region. After introducing the method of sequences, a generalization of Poincaré's balayage method, Professor Kellogg went on with the enunciation of an unsolved uniqueness problem, the solution of which is equivalent to showing that every bounded set of points of positive capacity contains a regular point. This point set theorem holds good in two dimensions and the question is whether it continues to hold in three dimensions. Dr. Kellogg discussed the capacities of Cantor sets and in conclusion raised the question whether the positive capacity of a point set is a topological property.

On Tuesday afternoon, Professor J. V. Uspensky of Stanford University spoke on *Integration in finite form*. He gave an ordered account of some classical researches of Abel and Liouville on elliptic integrals. He then discussed some of the arithmetic problems that arise in connection with Weierstrass' practical solution of the problem of finite integration by throwing it into a transcendental form with elliptic functions. He spoke of the periodicity of triads of integers and referred to Tchebycheff's criterion of periodicity (1860). In conclusion, he mentioned Zolotareff's remarkable criterion involving algebraic numbers (1874), and pointed out that Zolotareff devised his theory of ideal numbers as a by-product.

A. D. MICHAL,
Acting Secretary

GEORGE DANIEL OLDS

OCTOBER 14, 1853–MAY 10, 1931

With the death of George Daniel Olds, on May 10, 1931, the firmament of mathematical instruction lost one of its bright stars. The light he shed in and out of the classroom was always clear and steady, his guidance sure, and his influence inspiring and uplifting. Endowed with an exceptionally brilliant intellect and with a logical mind, he early became a seeker after truth. He sought to know where wisdom could be found and he seemed to discover the place of understanding. His life-long habit of reading and study brought him to a high level of attainment in a variety of fields, but mathematics was the interest nearest to his heart. It was with enthusiasm that he went abroad for graduate studies under such men as Fuchs, Quincke, Kirchoff, Helmholtz, and Weierstrass; and these masters recognized in their young American pupil a kindred spirit.

Listening to him as he taught his chosen subject, one could not fail to be impressed by the breadth of his background and the profundity of his knowledge. It became clear that he was above all a teacher, by virtue of his success in presenting mathematical truth in a manner adapted to the understanding of his students. At the same time, he imparted to them something of his own enthusiasm and love of learning.

Called in 1891 from his Alma Mater, the University of Rochester, he taught mathematics at Amherst College until 1923, when he was elected President of the College, a position which he held until his resignation in 1927. During his incumbency the duties of his office demanded an extraordinary degree of wisdom and tact, and the history of his successful administration will ever remain a worthy monument to his character and ability.

But remarkable as were his achievements as teacher and as president, it was his capacity for making friends that made him almost unique among men. During his forty years of residence in Amherst, including thirteen years as Dean of the College, the thousands who knew him would bear unanimous testimony to the justice and loyalty of George Olds, the man.

*"He had a genius for friendship; everybody felt it. There was no distinction of class, race, or color; no Town and Gown. The smile that warmed you when he greeted you, lifted you, whoever you were, to his plane, and made you an aristocrat with him. No wonder we loved him; he knighted us all with the touch of his character. His kind of friendship comes only from the deeps; it is no surface glitter. His own soul was so clear, so steadfast, so real, that he saw only those things in others. And his own steadfastness came from a strong faith, which was childlike in that it was clear-seeing and untainted, but which had for its foundation a very fine mind disciplined by training and hard work. '*Sans peur et sans reproche*'—without fear because he had nothing to hide, without taint because second-rate things were crowded out by his faith, courage, and vitality. As men pass, others are called to fill their places, but what one man can we find combining his fine training, his high purpose, his humor, his friendliness, and his faith in God? He was 'a verray parfit gentil knight.'"

T. C. ESTY

* From The Amherst Record.

LEFSCHETZ ON TOPOLOGY

Topology. By Solomon Lefschetz. New York (American Mathematical Society Colloquium Publications, Volume 12), published by this Society, 1930. 409 pp.

The rapid growth of interest in the science of analysis situs has been one of the striking facts of recent mathematical history. Of decided importance in the development of that branch of the science which deals with n-dimensional manifolds was the publication in 1922 of Veblen's Colloquium Lectures. In this volume the connectivity theory, one of the finest creations of Poincaré, was put onto the rigorous foundations which had been prepared for it in the work of Veblen and Alexander, and the many problems which the more exacting viewpoint entailed were clearly brought into evidence. The "speedy obsolescence" which Veblen wished for his book has by no means taken place; but the success of his work is evident from the many important advances to which it has given stimulus. One of the outstanding leaders of this recent progress is the author of the present volume. His researches have extended over a period of several years, and have now been brought to a brilliant culmination in this latest addition to the Colloquium Publication series.

Almost the whole of Lefschetz's work in analysis situs has centered around the following fundamental problem: a given space is subjected to a continuous transformation into itself; how can a topologically significant index be attached to the totality of fixed points and how can its value be computed in terms of the invariants of the situation? In an early series of papers on surface transformations, Brouwer included the first explicit solution of this problem for a very special case. If a sphere is subjected to a single-valued sense-preserving transformation into itself, then, as Brouwer pointed out, the algebraic sum of the multiplicities of the fixed points is always two, and from this the existence of at least one fixed point is immediately inferred. Other authors treated somewhat more general cases and a number of interesting results were obtained, notably by Birkhoff and Alexander. It was Lefschetz, however, who in 1923 proposed a method of attack which penetrated into the heart of the problem in its most general form, and brought it immediately within working range of the tools of n-dimensional analysis situs as they had been developed in Veblen's lectures. If M' is a copy of a given space M, let us form, said Lefschetz, the product space $M \times M'$. Then a transformation T of M may be thought of as a pairing off of the points of M and M', and the totality of point-pairs will constitute a subspace V of $M \times M'$. The identity in particular will lead to subspace V_0, and the fixed points of T will then correspond to the intersections of the two subspaces V and V_0. Thus the concept of transformation, with its connotation of change, is essentially a static geometric situation, and as such can most advantageously be studied.

This, then, is the guiding principle for Lefschetz's attack on the problem, and it is interesting to observe the fruitfulness of an idea of such simplicity and intuitive content. There is hardly an important situation which does not yield

to its power. Not only may the manifolds be of arbitrary dimension and structure, with or without boundary, but the transformations themselves may be of almost the greatest possible generality consistent with the word *continuous*. The author goes even further, and in a highly interesting chapter on infinite complexes, shows how his methods can be applied to arbitrary compact metric spaces. That a theorem such as the following is a very special corollary indicates the range of the results. Suppose a compact metric space to be simply connected in every order, as well as locally connected; then every continuous single-valued transformation of the space into itself has at least one fixed point.

It must not be supposed that the present volume deals exclusively with a single problem. On the contrary, it is rather a complete and skillful exposition of the analysis situs in which the problem is immersed, and this means nearly the whole of the theory of complexes as it now exists. The foundations of the combinatorial part are firmly laid at the outset. There follow the fundamental invariance proofs. In the chapter on manifolds, the introduction and systematic use of *combinatorial cells*, which like many other fundamental ideas in this connection are due to Alexander, is particularly noteworthy. For in terms of these cells it becomes possible for the first time to formulate a combinatorial definition of manifold which is topologically invariant. The author's proof of this, following somewhat along the lines of the first published one, that of van Kampen, is remarkably compact, thanks to the use of the author's convenient relative concepts, to which we shall again refer. In this same chapter, the author rounds out his recent interesting results with regard to duality relations. There are two well known types of such relations: those discovered by Poincaré which exist between the various connectivity indices of a manifold, and those due to Alexander in which the invariants of a surrounding residual space also enter. The author's discovery that these two types of relations are special cases of a third more general type, is revealed in a set of formulas of striking symmetry and generality.

The remainder of the book contains the material which is most closely connected with the author's own researches,—the theory of intersections and Kronecker indices, product complexes, fixed point and coincidence formulas. The majority of the results here are the author's own, and many of them are already in the literature. In its complete systematic development, however, this part of the theory has been considerably strengthened; the results are now more general and many of the proofs much simpler. In order to carry over the main body of his results to a much more general class of spaces than he had previously considered, the author has included a new general theory of infinite complexes. The fact that an arbitrary open n-dimensional region is a very special type of infinite complex, is an indication of the types of spaces which this theory brings into play. The author obtains a complete classification of the various sorts of homology bases which can occur in the infinite case, and he is able to extend his earlier duality theory by the ingenious introduction of "ideal" elements. By means of the far reaching approximation methods of Alexandroff and Vietoris we are led to a homology theory for arbitrary compact metric spaces, and it is at this point that the duality and transformation theories receive their final extensions.

The book closes with a summary of the principal applications of analysis

situs to function theory and geometry. It is no mere accident that the foregoing theory can be so admirably applied to the solution of certain difficult problems in algebraic geometry,—those, for example, which concern the enumeration of the intersections of algebraic varieties and coincidences of algebraic correspondences. To some extent it is for these very problems that the theory has been created. They have especially influenced the author's mathematical work. He is known as an important contributor to algebraic geometry, and he writes here with particular authority. There are included also the applications to vector distributions, complex spaces, and the Kronecker characteristic theory. In all these questions, it is of course important to know that the analytically defined manifolds with which one has to deal are complexes in the technical sense,—that is, capable of being subdivided into cells. It has long been regarded as certain that such is the case, although, except for the relatively simple algebraic case, no complete proof exists. In sketching a possible proof for the general case, the author shows that he himself has no doubts about the matter. It may be remarked, however, that there is even less doubt that a thorough exposition of the details involved is a formidable task in function theory, and one that will be thoroughly worth undertaking.

From a technical point of view, the book contains much that is new and valuable. The simple device of neglecting, in a set of boundary relations, all elements which lie on a given configuration L, thus giving rise to relations "mod L," more than justifies itself by its usefulness, and will undoubtedly become a standard part of the combinatorial technique. By means of this device we are led immediately to the concept of relative invariants, and it is in terms of these that many of the author's most striking formulas are expressed. Moreover, it becomes unnecessary ever to treat separately the manifolds with boundaries and the theorems concerning closed manifolds now become corollaries of the more general ones. Worth noting also is the neat explicit statement of the fundamental deformation theorem and its various extensions. This theorem was implied in Alexander's original proof of the invariance of the homology characters and again in the same connection by Veblen in his Lectures. Its utility is almost universal, for in any given topological situation it permits us to limit our attention to the configurations composed strictly of the cells of the given spaces. How extensively Alexander's elegant new technique will be applicable, remains still to be seen. The present work, however, makes it quite clear that the older deformation method, on account of its simplicity and intuitiveness, is not likely soon to outlive its usefulness.

In the matter of presentation, we feel that the author has done an excellent piece of work. In a subject where the formal element plays so small a part, the writing is often difficult. While Lefschetz's exposition is necessarily compact and requires close attention on the part of the reader, it will be found to be accurate* and essentially complete. It adds no little to the work that the various

* The book is remarkably free from misprints. The following minor corrections have been suggested by the author:

page 3, line 11, replace "of R" by "for $T \cdot R$"
" 80, " 3, " "$T \cdot C$" by "$T \cdot C$"
" 91, " 18, 19, " "regular" by "normal"
" 122, between 3d and 4th lines from bottom, insert: (see page 648)

clerical details which can be of considerable aid to the reader,—the careful numbering of paragraphs and formulas, adequate cross-references, and the explicit statement of theorems,—have been scrupulously attended to. There are numerous diagrams and an excellent bibliography.

Analysis situs is a comparatively young science; yet its importance can not be doubted. It deals with the most primitive questions of geometry and is fascinating to those who enjoy moulding in precise mathematical rigor the visions of a sharpened physical intuition. Its most important problems deal with the very structure of space, and many of them are still to be solved. It is pleasant to reflect that much of what has been accomplished has been the work of American mathematicians, and to that work the present volume is a distinguished contribution.

P. A. SMITH

FUBINI AND ČECH

Introduction à la Géométrie Projective Différentielle des Surfaces. By Fubini et Čech. Paris, Gauthier-Villars, 1931. vi+291 pp.

This volume is in some sense a sequel to the treatise entitled *Geometria Proiettiva Differenziale* published in two volumes in 1926 and 1927 by the same authors. It should receive a generous welcome from the geometrical public for several reasons. First of all, it is in French, a language admittedly more widely read than Italian. The authors, profiting no doubt by their previous experience, have produced a quite readable book. Some detailed developments of their treatise have been omitted; the treatment here is more elementary, and the style of exposition is clearer than before. The discussion is confined to surfaces in ordinary space. Altogether, this book should serve well its purpose of being an introduction to projective differential geometry.

It must not be understood that the present volume is merely an abstract from the treatise. In fact, certain subjects are included which do not appear in the larger work at all. An analysis of the contents of the volume under review will amplify these remarks.

There are in all fourteen chapters, of which the first three may be regarded as introductory. The first is properly an introduction, containing some preliminary analytical results concerning collineations and correlations, matrices, and algebraic forms. The second treats of plane curves, and the third of curves in ordinary space.

Chapters IV–VIII contain an exposition of the projective differential theory of curved surfaces in ordinary space. The point of view is primarily that of the method of differential forms, but the differential equations which define a

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(Footnote contined from page 647.) "The proof as outlined holds for simplicial and convex cells, the only types for which it is used later in the book. The proof for the general case has been obtained recently by A. W. Tucker."

Chapter III, No. 45, replace everywhere "M" by "$\overline{M}$."

page 206, end of No. 49, add, "For $n=1$, $Lc\,(\Gamma_0 \cdot \Gamma_0')$ can also be defined provided that either Γ_0 or $\Gamma_0' \sim 0$.

page 403, end of reference to W. Mayer, add "219–258."

surface except for a projective transformation are also employed. The asymptotic curves are taken as parametric throughout.

The next three chapters are comprised in a separate group. Chapter IX is devoted to ruled surfaces. In Chapter X we find an account of the theory of plane nets. The well known theorem of Koenigs, that the perspective of the asymptotic curves on a surface in ordinary space from a point onto a plane is a plane net with equal Laplace-Darboux invariants, and conversely, suggests that plane nets with equal invariants should be called *asymptotic plane nets.* These nets are studied in some detail, particularly with reference to the perspectives of the various configurations associated with a point of the surface. The research in this chapter is, for the most part, guided by the idea of associating with a point of a plane net configurations analogous to those associated with a point of a surface, and more generally by the idea of the analog between a curved surface and a flat surface. Chapter XI is designed to present some results on point-correspondences between two surfaces.

The last three chapters are given over to the methods of Cartan. Chapter XII contains a résumé of the analytic theory of equations of Pfaff, and is preliminary to the final two chapters, which are taken up with an extended exposition of the methods of Cartan and their applications to the projective differential geometry of surfaces.

The reader who is familiar with the treatise previously referred to will see from the foregoing analysis that the last five chapters of the volume before us contain material not found in the two-volume treatise. These five chapters extend over 110 pages, or approximately two-fifths of the total number of pages.

There are two features of the new book which deserve to be especially commended in conclusion. The first of these is the excellent set of historical and critical notes at the ends of the chapters. These notes contain various comments and references to the literature. The second praiseworthy feature is the bibliography at the end of the text, containing about 600 references to books, memoires, and notes. This is probably the most ambitious bibliography of projective differential geometry yet published, and is an impartial scientific effort to construct a scholarly bibliography of the subject. However, it is not perfect. For example, the authors do not recognize that the brothers A. L. Nelson and C. A. Nelson are two different men, and credit C. A. Nelson with papers of A. L. Nelson. The reviewer would like to suggest that there still seems to be room for some one to write a comprehensive history of projective differential geometry, and to compile a bibliography complete to date to accompany it.

E. P. LANE

SHORTER NOTICES

Encyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Volume 3 in three parts. Geometry, edited by W. Fr. Meyer and H. Mohrmann. First part: A. *Rein geometrische Theorien.* B. *Grundlagen der Anwendung von Algebra und Analysis auf Geometrie.* Second half, articles 8–13. Leipzig, Teubner, 1914–1931. xii+769–1595+1–264 pp.

This volume appeared in the following six parts, at the times indicated.

Heft 5, June 1914, contains article III AB 8 by J. Sommer, *Elementare Geometrie vom Standpunkte der neueren Analysis aus*; and part I of article III AB 9 by M. Zacharias, *Elementargeometrie und elementare nicht-euklidische Geometrie in synthetischer Behandlung.*

Heft 6, January 1921, contains part II of article III AB 9.

Heft 7, January 1921, contains article III AB 10 by G. Berkhan and W. Fr. Meyer, *Neuere Dreiecksgeometrie*; and part I, by H. Rothe, of article III AB 11, *Systeme geometrischer Analyse.*

Heft 8, July 1924, contains part II, by A. Lotze and Chr. Betsch, of article III AB 11.

Heft 9, May 1922, contains article III AB 12 by E. Steinitz, *Polyeder and Raumeinteilungen.*

Heft 10, December 1930, contains article III AB 13 by H. Tietze and L. Vietoris, *Beziehungen zwischen den verschiedenen Zweigen der Topologie.*

An index for the entire part I of Band III appears at the end of the volume, by A. Boy. Articles 12 and 13 have their pages numbered 1–264, in italics, evidently because part II of article 11 appeared two years after article 12.

The subjects covered can be assigned roughly to three categories. Most of the volume (articles 8, 9, 10, 12) is devoted to topics such as ordinary euclidean geometry, and geometry of configurations in euclidean spaces. Another division of the volume consists of the two parts of article 11, on systems of geometric analysis, by Rothe, Lotze and Betsch. This section, covering pages 1277–1595, deals with a wide variety of subjects of great interest to analysts and mathematical physicists, as well as to geometers. Finally, there is the Tietze-Vietoris article on topology, the only recent article in the volume.

The Dehn-Heegaard article on *Analysis Situs* (III AB 3) deals largely with formal combinatorial operations, which have never been proved to correspond to general homeomorphisms. The Zoretti-Rosenthal article *Die Punktmengen* (II C 9 a) contains a considerable amount of material on general topology, for the most part in the small, but makes only few direct references to the complex of analysis situs. The Tietze-Vietoris article at the end of the present volume is intended to be a connecting link between the latter two articles. It gives very full discussions of some of the fundamental ideas and problems of analysis situs, and covers well the many important developments of the subject in recent years. Numerous references to American mathematicians bear witness to their important contributions to topology in the last two decades.

A. B. Brown

Spektraltheorie der unendlichen Matrizen. Einführung in den analytischen Apparat der Quantenmechanik. By Aurel Wintner. Leipzig, S. Hirzel, 1929. xii+280 pp.

In the Introduction the author defines the main purpose of the book (not quite in accord with the subtitle) to serve as an introduction to the general theory of linear analysis of infinitely many variables. It is stated that, according to the desire of the publishers, the book is designed primarily for beginners. This desire led the author to some compromises as to the choice of material and the character of exposition, of which the most regrettable, although perhaps not entirely unavoidable, is the exclusion of the notion of Lebesgue's integral.

Chapter I is devoted to a rapid survey of fundamental facts of the theory of matrices and bilinear forms in a finite number of variables. We find here a condensed treatment of reduction of matrices to various canonical forms with applications to the theory of hermitian, unitary and normal matrices, of Jacobi's transformation of matrices and its application to hermitian matrices with a simple spectrum. As the author expresses himself, the material is presented here, not always in the simplest and most natural way, but rather in such a way as to permit extension almost without modifications to the theory of infinite matrices. While much might be said in favor of such a method of presentation, it can be hardly considered as the best one for the beginners. Only an experienced and well informed ("kundiger") reader can appreciate many a subtle detail which would puzzle a beginner.

In Chapter II the author gives a discussion of indispensable analytic tools: Stieltjes integrals, properties of sequences of functions of bounded variation (theorems of Helly and Helly-Bray), inversion formulas of Stieltjes and Hilbert, theorems of Grommer and Hamburger. At the end of the chapter the author attempts to give a "gemeinverständlich" report on Hellinger's integrals. No adequate idea of this theory can be given without using the notion of measure, which is being carefully avoided by the author. Hence, in the reviewer's opinion, the book would only gain if the corresponding pages, 106–120, had been omitted.

Chapter III deals with general properties of bounded matrices and of their resolvents: "Faltung" theorems of Hilbert, criteria and "formal" theorems of Toeplitz, theorems of Hellinger and Toeplitz, C. Neumann's series for the resolvent, characterization of the resolvent as an analytic function of the parameter. The treatment is elegant and presents several novel points of interest.

Chapter IV gives a rather condensed and somewhat incomplete discussion of the spectral matrices. A "beginner" will not readily understand the "Hauptsatz über Einzelmatrizen" on page 159 and the subsequent discussion, even if the formula (258) on page 160 did not contain a disturbing misprint (compare with (76) on page 53).

Chapter V is devoted to the existence proof of spectral matrices for various classes of bounded matrices. This chapter contains material of considerable interest and importance. Several results of this chapter are due to the author, among them the elegant treatment of the unitary matrices and their spectral matrices on the basis of the trigonometric moment problem. The reviewer was not able, however, to follow the proof on page 174.

In Chapter VI we find various extensions of previous results to certain hermitian non-bounded matrices. Some of Carleman's results are interpreted from the point of view of matrices. A relationship with the Stieltjes-Hamburger moment problem is briefly indicated.

In the Appendix the author gives a rather brief sketch of his own investigations in the spectrum theory of the almost-periodic functions of H. Bohr.

From the above enumeration, which is necessarily rather incomplete, it is seen that the book in question contains interesting and important material which is partly new or is treated from an original point of view. When reading the book one can not help feeling, however, that the author endeavored to put too much material in too restricted a space. The result is that the author has not completely succeeded in writing an "introduction" to a great theory, which could be used to advantage by a "beginner." As to a well informed and experienced reader, it might happen that the latter will feel better off when he turns to the original memoirs, including some by the author himself (not to speak of papers of J. v. Neumann, which have appeared simultaneously with the publication of the book). One point deserves to be mentioned separately. There are found in the book about three dozen new terms introduced by the author; here are some of the most striking ones: "Hellysche Fortpflanzungssatz," "quadratically convergent vectors," "infinitesimale und integrale Integrabilitätsbedingungen," "statistisch sinnvolle Matrizen," "wasserstoffähnliche Spektra," "Carleman's Feldtheorie," "Stabilität des reducierten Spektrums," "Hellinger function-pairs of the first and second kind," etc. Not all, and even not most, of these new terms correspond to actually new notions, and many of them do not serve to describe the situation in the best way. For instance a Hellinger function-pair of the second kind simply means a pair of functions $\rho(\mu)$, $\sigma(\mu)$, continuous, bounded and not decreasing on $-\infty<\mu<\infty$ and approaching 0 as $\mu\to-\infty$. If, however, the introduction of a new term is unavoidable, it seems desirable that its definition should be stated in precise expressions, which in many cases has not been done in this book. The fact that not one of the theorems or definitions is underscored or separated from the body of the text will also contribute somewhat to the discomfort of the reader, particularly of a beginner. The bibliography of the subject is given "en bloc" at the end of the book, without explicit references in the text; this is not always convenient. Finally, misprints and slips of the pen are not infrequent.

J. TAMARKIN

A Course of Geometrical Analysis. By Haridas Bagchi. Calcutta, Chuckervertty, Chatterji and Company, 1926. iv+562 pp.

This rather long book on elementary differential geometry is written by a Premchand Roychand Scholar and Lecturer in Mathematics in Calcutta University. It gives evidence of wide reading and of much thought and study. While for the most part the topics and the treatment follow classic lines, there are many discussions evidently original with the writer. It is apparently intended as a textbook, for there are frequent references to the student and suggestions offered to him. It is, however, curious in arrangement, widely discursive in treatment and remarkably uneven in difficulty; it does not seem to us so well adapted for one beginning the subject as the familiar French, German and

American texts. The author is of the English School, as appears both from his references, chiefly to Forsyth, and from his methods of proof. He is interested more in geodesics than in any other topic, as may be seen from his devotion of over one hundred pages to the subject. Towards the end of the book there is occasional use of vector methods. There are a number of discussions of problems of algebra and mechanics, some extraordinarily elementary.

In other texts with which we are familiar, the general theory of curves is studied before the special "organic" curves of surfaces, that is to say the minimal lines, the asymptotic lines and the lines of curvature; then some simple surfaces, for example surfaces of revolution, are studied before the general theory. In this book it is not so; an idea of the arrangement is given by the titles of the first six of the eight chapters: I. Organic Curves of a Surface, II. Genesis of the Fundamental Magnitudes, III. Choice of Parameters and Geodesic Parallels, IV. General Theory of Geodesics, V. Surface of Revolution, VI. General Theory of Curves.

The book is remarkably free from errors and misprints.

J. K. WHITTEMORE

Leçons sur les Systèmes d'Équations aux Dérivées Partielles. By Maurice Janet. Paris, Gauthier-Villars, 1929. viii+122 pp.

This book (volume IV of the interesting new collection "Cahiers scientifiques" edited by G. Julia) is a welcome introduction to the general theory of systems of partial differential equations. In the preface the author proposes two standpoints from which the subject might have been treated, the analytical one and that of mathematical physics. Only the former of these two points of view is given consideration in the book. It deals with analytic functions of complex variables and can be characterized as being purely local, while the latter is interested primarily in real variables and prescribes in advance the domains in which the solutions in question have to be determined. That such a classification is somewhat artificial is clearly shown by several recent investigations where the fusion of the Cauchy and Dirichlet problems has been used to great advantage. It also led the author to exclude from consideration many an important problem of the general theory of partial differential equations. Such an exclusion, however, was wise lest the number of pages should increase beyond a reasonable limit. Anyhow, the author has succeeded in giving an interesting and easily accessible exposition of the Cauchy problem for general systems of partial differential equations in any number of unknown functions. A large number of examples and exercises help the reader considerably in mastering the subject, whose main difficulty is of algebraic rather than of a purely analytic nature. At the end of the book we find a "Bibliographie sommaire" which contains several references to the author's own work (7 out of a total of 15). The fundamental work of N. Günther who, together with Ch. Riquier, should be considered as a main contributor to the subject, deserves more than a reference to a Comptes Rendus note and a short footnote (1) on page 119, where the important doctor's thesis of Günther* figures under the heading "nombreux travaux en langue russe."

J. TAMARKIN

**On the theory of characteristics of systems of partial differential equations,* St. Petersburg, 1913, xiv+378 pp. (in Russian).

The Elementary Theory of Tensors. By Tracy Yerkes Thomas. New York, McGraw-Hill, 1931. 10+122 pp.

This text presents the elements of geometry and mechanics in tensor notation. It will in consequence be useful either as a basis for a modernized course in vector analysis or as a reference for the undergraduate who delves into popular Einsteiniana and asks his teacher "What are tensors?".

The argument is well and carefully thought out. In particular, the fundamental equations of euclidean and kinematical transformations are derived by methods not readily available elsewhere. It is perhaps a regrettable consequence of the choice of applications that, with the exception of the vector product, all the physical or geometrical examples of tensors mentioned are first-order tensors, that is, vectors. Besides a few harmless misprints the reviewer noted only two mistakes. The footnote on page 68 implies that every rigid displacement of the plane is a rotation with finite center, although the close reader will recall the mention in the text of the case in which the center recedes to infinity. On page 37 we read "No legal standard of length has been adopted by the United States and in the absence of such a standard the British system of measurement has come into practical use."

P. FRANKLIN

Vorlesungen über die Singulären Moduln und die Komplexe Multiplikation der Elliptischen Funktionen. Part II (Teubner's Sammlung, Band XLI, 2). By R. Fueter. Leipzig, Teubner, 1927. 143–358 pp.

This is the second and concluding volume on the subject by Professor Fueter, the first of which appeared in 1924 and was reviewed by Dresden in the American Mathematical Monthly (vol. 32 (1925), pp. 474–476). The reader is urged to consult the Dresden review for orientation. Let us recall that in Part I the author has developed, compactly, the theory of the modular group and associated functions, and the algebraic-arithmetic theory of quadratic number fields, including ideal-theory and the many concepts stemming from this important branch of mathematics.

In Part II Professor Fueter enters the deeper realms of his subject, giving particularly the important extensions since Weber's *Algebra*, vol. III. While most of the book is arithmetic and algebraic in character, analytic methods are by no means eschewed. Thus, in the course of the volume, there is a much-trodden path leading from the elliptic function addition theorems, and the author, travel far as he might in the higher theory of number fields, never loses contact with these fruitful formulas. In addition, we find on a few occasions that the zeta-functions of Dirichlet and Dedekind are utilized to establish a theorem neatly. The author has given in his preface a two-fold explanation of the advantages of function-theoretic methods; algebraists will be interested in the points that he makes.

The present volume is a continuation of the other both in chapter numbering and paging. Thus we begin with Chapter VI, on the factorization of prime ideals. It should be recalled that the fundamental number fields in this theory are the quadratic imaginary number fields $k(\sqrt{m})$ (or simply k). With respect to k one considers larger containing fields K, and in particular such fields K as have, in k, an abelian group. A field of this character is a relative-abelian field

(with respect to k). Important examples are *class fields* and *ray class fields*, introduced in vol. I. The present chapter is devoted to the proof of a group of theorems on the factorization of prime ideals in both class fields and ray class fields. We find here the first use (in the book) of the zeta-functions; and by their means it is established that the ray class equation is irreducible. On the other hand, the class equation is shown to be factorable upon adjoining to k a certain number of square roots. A similar theorem pertains to the ring class equation.

Chapter VII studies in detail the general ray class field and its relative discriminant. First there is a careful treatment of those ray class fields whose leaders are multiples of the ideal (4), after which the general case is handled. The theorems of this chapter relate to the composition of ray class fields, the nature of the prime ideals that are contained in the relative discriminant of $K(4)$, and the *Relativdifferente* of a ray class field. In particular, if f is an ideal of k, then the ray class field determined by choosing f as a leader is relatively Abelian with respect to k. Finally, for the general ray class field there is a fundamental theorem (pages 262–263) which is an excellent summing up (space and technicality of language do not permit quoting); a summing that brings one out of the maze of details and proofs onto a summit from which one can make a survey of the next problem.

That problem is the proof of *completeness*, which, in the words of the author, has been the goal of the whole book and is the crown of the entire theory. What is this theorem of completeness, which holds, and rightly, so high a place in the author's esteem? It can be put in different forms; one of the most striking is this. *Every abelian equation in a quadratic imaginary field* $k(\sqrt{m})$ *can be resolved by means of singular moduli and singular elliptic functions.* (These singular quantities are defined in volume I.) The first proof of this fine result appears in the present volume.

There is a Chapter IX on the computation of the singular values of a certain modular function and of associated equations. A good part of this chapter is function-theoretic in character. The problem of computation is by no means a simple one, as any reader will see.

So much for the mathematical content of the book; and that stands high indeed. What of the personal content: the style, the ease with which the material can be grasped? It has been intimated that the book is written very compactly; and so it is. There are, for example, as many as 254 theorems in the space of 305 pages (8 chapters), and most of the proofs are by no means trivial. This suggests that perhaps a student would be carried from page to page, from theorem to theorem, so intent on mastering proofs as to miss the important stations on the journey. And indeed, a first study of the volume is likely to be so characterized. But, we can acknowledge, a second reading brings topographic "relief": the high points *do* stand out, and the journey through the book, (to the original contents of which Professor Fueter has notably contributed), is a fascinating one. Certainly, experienced algebraists will find the book profitable and important, as a noteworthy extension of knowledge in this field. To those with but a modicum of knowledge of the subject matter, the book will still be fascinating; but, let us admit it, for such the road will not be a royal one.

I. M. SHEFFER

Cours de Mathématiques Générales (Analyse et Géométrie). By R. Garnier. Vol. 2. *Calcul Intégral*. Paris, Gauthier-Villars, 1931. 6+396 pp.

The first chapter of this volume (Chapter 12 of the two-volume course) is devoted to the exposition of integration of continuous functions. The idea of definite integral is introduced before that of indefinite integral. Chapter 13 deals with double integrals and triple integrals. Chapter 14 takes up line integrals and surface integrals in the customary manner, with some vector analysis.

Chapter 15 is on ordinary differential equations. The discussion is elementary, yet more complete than is usual. Among other topics we find: singular integrals, limiting curves, and the delimitation of regions of a plane covered by integral curves associated with different analytic forms of integrals.

Chapter 16 is on linear partial differential equations. The geometric discussion of first-order equations is quite clear. The higher order equations are illustrated by the equation of the vibrating cord, with the d'Alembert and Bernoulli solutions. Several examples of Fourier series terminate the chapter.

There are six notes at the end of the volume: oblique coordinates, systems of sliding vectors, envelopes of straight lines in normal form, practical decomposition of rational functions with complex poles, explicit integration of certain indefinite integrals by means of inverse circular or hyperbolic functions, the correspondence between circular and hyperbolic functions.

There are three obvious misprints: page 13, line 25; page 38, line 2, and page 208, line 3. The author is to be commended for his insistence upon the precautions to be taken in changing variables in integration, especially in the choice of proper determinations of inverse functions. The treatment of systems of ordinary linear differential equations with constant coefficients might have been more complete. The material of Chapter 15 (110 pages) supplemented by a goodly number of problems would make an interesting course for American undergraduates: the other chapters are covered, in general, in many of our advanced calculus texts.

W. E. Byrne

Darstellung und Begründung einiger neuerer Ergebnisse der Funktionentheorie. By Edmund Landau. Second Edition. Berlin, Julius Springer, 1929. 122 pp.

This new edition of Landau's well known work is a welcome contribution to modern literature on the theory of functions. It is by no means a mere reprint of the first edition (1916). Some new material has been added, some proofs simplified, and in fact, as Landau points out, one fourth of all the references are to papers which have appeared since the publication of the first edition.

The book treats a number of selected topics in the theory of functions of a complex variable, particularly topics which either in subject-matter or method are concerned directly with power series, topics to which Landau himself contributes definitely in discovery or presentation. The book does not claim to include all the principal results of modern function theory; for instance the names of Jensen, Julia, Montel, and Ostrowski do not appear.

Chapter I is concerned with limited analytic functions, and has been enriched since the first edition by the addition of Carathéodory's proof of Fatou's theorem, namely that a function analytic and limited in the unit circle C pos-

sesses boundary values for radial approach to the circumference almost everywhere on C. This is a useful addition, for Fatou's theorem is of great importance, for instance, in proving the validity of Cauchy's integral formula for the function or in the study of conformal mapping, and Carathéodory's proof is entirely elementary, not requiring even the formal use of Lebesgue integration.

Chapter II, on summability, is devoted chiefly to a proof of the Knopp-Schnee theorem of the equivalence of the methods of Cesàro and Hölder. Chapter III is on the converse of Abel's theorem relative to the continuity of the function represented by a power series, the main result being due to Hardy and Littlewood. Chapter IV contains some specific examples to illustrate various possibilities as to the nature of the convergence of a power series on the circle of convergence.

Chapter V studies the connection between the coefficients of a power series and the singularities of the function represented; it contains particularly M. Riesz's theorem that if $a_n \to 0$, the power series $\sum a_n x^n$ converges in each point of the unit circle of regularity of the function represented, and the convergence is uniform on any closed arc of regularity of the function; and Fabry's theorem on singular points.

Chapter VI is on the maximum and minimum values of the modulus of an analytic function. Hadamard's three-circle theorem is proved and applied to the proof of Jentzsch's theorem. The proof by Pólya and Szegö of Hardy's theorem on the monotonic character of

$$I(r) = \int_0^{2\pi} \left| f(re^{i\phi}) \right| d\phi$$

is also given. It would have been worth while to have given the corresponding result (perhaps also as proved by Pólya and Szegö) for the more general function

$$I_p(r) = \int_0^{2\pi} \left| f(re^{i\phi}) \right|^p d\phi, \qquad (p > 0).$$

Chapter VII treats Picard's theorem on essential singularities, together with the related theorems of Bloch, Landau, and Schottky. Chapter VIII is the only one dealing with conformal mapping as such, although Chapter III gives Fejér's theorem on the convergence of the power series for a smooth (schlicht) function, and gives particularly the proof of Koebe's theorem on distortion.

The entire work is written in Landau's characteristic style—concise, clear, complete. The proofs are simple and are carried through in full detail, and the formulation, correlation, and presentation of such proofs represents the most important service rendered by the book. The reviewer could wish that the supplementary remarks were more elaborate at times, that motivation for some of the proofs was given, and that there were here and there hints of elaboration and application of results, carried out in detail elsewhere in the literature. Such additions would make the book more valuable for the beginner. But the book is already valuable for the beginner, and is indispensable for the more advanced student.

J. L. WALSH

Differentialrechnung. By A. Witting. Berlin and Leipzig, de Gruyter, 1931. 191 pp.

This volume is Number 87 of the Sammlung Göschen and replaces the earlier one on differential calculus. It covers essentially the same ground as a half year introductory course in American colleges, with about the same degree of rigor and the same applications. The 185 exercises in the text are drill problems to which answers are given. This book maintains the high standard of printing and appears in the same convenient pocket size and durable linen binding as other recent volumes of this collection.

W. R. LONGLEY

Projective und Nichteuklidische Geometrie. By F. Schilling. Leipzig and Berlin, B. G. Teubner. Volume I, xi+212 pp.; Volume II, xi+216 pp.

The principal object of Professor Schilling's treatise is to lay the foundations and derive the important features of projective geometry in a systematic manner independent of euclidean metrics, and to establish non-euclidean geometry upon a projective basis as it had been accomplished before by his illustrious teacher Felix Klein. The result is a very broad and thoroughgoing treatment of the whole subject which the student can use with much profit. In the first volume Schilling makes the passage from the synthetic to the abstract analytic formulation in a very illuminating manner by means of the dyadic number system involved in Möbius' harmonic quadrangle construction. This is undoubtedly the simplest and fastest method of approach in the establishment of a system of projective coordinates. It seems to the reviewer that Schilling's treatment could be still more simplified by making use of the fact that every real rational or irrational number can be represented as a dyadic number. Thus by a continuity axiom and eventual limiting process every number or point is obtained in the constructive scheme.

In the study of collineations particular attention is given to those that leave a fixed conic invariant. The purpose of this is of course to prepare the way for non-euclidean geometry in the second volume. One also finds a very interesting section on the birational mapping of the projective plane upon a closed surface without singularities in a limited portion of space.

Volume II contains a very satisfactory exposition of non-euclidean geometry from a projective stand-point in which the groups of movements in the three types of geometry (including euclidean) assume a fundamental role. Schilling does not of course fail to show the connection of the projective standpoint with that of Riemann's "Habilitationsschrift." He also gives an ample treatment of non-euclidean trigonometry, so that altogether the book forms a welcome addition to the projective treatment of the subject.

The reviewer has noticed very few errors. On page 3, volume II, theorem 2. "Eindeutigkeitsaxiom" of *plane* movements contains an inaccuracy. In Fig. 1, it is obvious that the shaded sides cannot be assigned arbitrarily as corresponding, as would be possible by the statement in the text. The typography and general make up of the books are excellent.

Their value is furthermore increased by a large number of literary references and comments.

A. EMCH

A Simplified Presentation of Einstein's Unified Field Equations. By Tullio Levi-Civita. Authorized translation by John Dougall. London, Blackie and Sons, 1929. 22 pp.

It should be stated at the outset that the author of this pamphlet does not merely elucidate the work of Einstein but attacks the problem *ab initio*. For the specification both of metrical space and of electromagnetic phenomena he uses, (as did Einstein), the sixteen parameters, λ_i^α, $\alpha=1, \cdots, 4$, defining four orthogonal congruences, $i=1, \cdots, 4$. ($\lambda_i^\alpha = {}_i h^\alpha$ in Einstein's notation.) The ten coefficients $g_{\alpha\beta}$ in the expression for ds^2 are completely determined by these parameters, but not conversely. For the complete determination of the parameters ten relations are furnished by the usual Einstein gravitational equations; the remaining six relations come from Maxwell's equations in connection with a (postulated) relation between the electromagnetic field tensor and the parameters. The author avoids the use of "covariant derivatives with respect to the quadruplet" by "making use of perfectly familiar methods of the absolute differential calculus." The necessary fundamentals for a comfortable reading of this pamphlet will be found in Chapters 4–6 and 10–11 of *The Absolute Differential Calculus* by the same author.

C. A. SHOOK

Introduzione alla Geometria Algebrica. By Edgardo Ciani. Padova, A. Milani, 1931. x+599 pp. Mimeographed. Price Lire 60.00.

The present volume is a continuation of the author's *Metodo delle Coordinate Proiettive Omogenee* (2d edition, Torino, 1928). Thus it presupposes a general knowledge of projective coordinates, of determinants and the elements of invariants, and of the processes of the calculus. It is primarily of didactic value; all proofs are given, and no sources or citations to other methods are given. The book has no figures and is not provided with an index.

The first thing considered is the binary biquadratic form, its two invariants, its covariants, the harmonic and equianharmonic forms, and the syzygetic involution. Then follows a very brief discussion of algebraic plane curves, including Plücker's numbers, the hessian, and points of inflexion, and a similar one for surfaces and space curves. Quadratic (Cremona) correspondences between two planes with a brief treatment of the resolution of singularities of plane algebraic curves precedes the more systematic discussion of quartic curves in two and in three dimensions. A fairly full chapter on cubic surfaces is followed by one on selected quartics, particularly the Kummer surface and the Steiner surface. Much of the author's own work is given in the chapter on geometric applications of the theory of groups. Finally, about one-fourth of the volume is devoted to properties of four-space. It includes the representation of points, lines, planes and three-spaces by equations and parametrically, collineations and correlations, then an outline of quadratic varieties and of the cubic variety.

The mechanical preparation of the volume is excellent, and makes it particularly easy to read. As a general introduction to the field, the book accomplishes the purpose in a satisfactory manner, but it would have been much more valuable had a bibliography been added.

VIRGIL SNYDER

Leopold Kronecker's Werke. Volume V. Edited by K. Hensel. Leipzig, Teubner, 1930. 10+528 pp.

The edition of Kronecker's works is now rapidly nearing its completion; there remains only Part 2 of Volume III yet unpublished. According to the list of Kronecker's publications compiled at the end of the present volume the total number of papers amounts to 139, a number which reveals the prodigious task of the editor. It seems the proper moment to congratulate Professor Hensel upon this achievement; he has by this edition erected a monument to the mathematical genius of Kronecker, a mathematician whose ideas have placed their stamp on the development of numerous branches of modern mathematics. Professor Hensel won his own first mathematical laurels by his completion of Kronecker's theory of algebraic numbers.

The present volume contains the second part of Kronecker's papers on elliptic functions and minor papers on function theory, potential, and mathematical physics. Then follows a set of shorter notes on a great variety of subjects ranging from number theory to physics and reflecting the wide interests of Kronecker. Furthermore there are Kronecker's published letters to various mathematicians, addresses and minor statements. Among the latter it seems to me that a few might well have been omitted, for example the list of the lectures of Jacobi compiled by Kronecker; the editor seems to have wanted, for the sake of completeness, to include all unpublished articles.

Among the letters I shall only mention the famous letter of Kronecker to Dedekind concerning the so-called "Jugendtraum" of Kronecker, that is, the representation of the numbers of a relative Abelian field with respect to a quadratic imaginary field by means of singular moduli. In some special annotations to this letter, H. Hasse discusses the possible formulations of this theorem; it is known according to the results of Fueter that if the field is of odd degree all its numbers can be expressed by means of roots of unity and singular moduli; for even degrees some additional irrationalities are required. Hasse points out that it is possible that Kronecker more generally has intended to use the irrationalities defined by the transformation of the singular elliptic functions, and that various of Kronecker's statements seem to indicate this. For such a formulation the theorem is always true as has been shown by Takagi. It is, however, highly probable that Kronecker never arrived at an exact formulation of his "Jugendtraum."

O. Ore

NOTES

On recommendation of the Council, the Board of Trustees of the American Mathematical Society has concluded a reciprocity agreement with the Unione Matematica Italiana by which the members of each organization have certain privileges in the other. The arrangement, which closely parallels that made with the London Mathematical Society a few years ago, has been made through President S. Pincherle of the Unione. On the one hand, members of the Unione not resident in the United States or Canada may become members of the American Mathematical Society by payment of the initiation fee and half the annual dues. On the other hand, members of the American Mathematical Society not resident in Italy may become members of the Unione without initiation fees on payment of half the annual dues. The rate for such persons is twenty Italian lire yearly. Members of the Society who wish to take advantage of this arrangement by becoming members of the Unione should send their names to the Secretary, 501 West 116th Street, New York, N.Y., who will forward them to the Unione.

The preliminary official program of the International Mathematical Congress to be held in Zurich in 1932, which has just been issued, indicates that the sessions of the Congress will be on September 4–12, that all countries will be represented, and that titles of papers intended for presentation at the Congress should be sent to the Secretary of the International Mathematical Congress, École Polytechnique Fédérale, Zurich. The President of the Committee on Organization of the Congress is Professor R. Fueter. It is the present intention also to hold the sessions of the International Mathematical Union, which is a distinct organization, and of which Professor W. H. Young is the President, during the time of the Congress.

The second International Congress of the History of Science and Technology was held at London, June 29–July 4, 1931, under the presidency of Dr. Charles Singer, of London. The vice-presidents were Professor Gino Loria, of Genoa, and Dr. George Sarton, of Harvard.

The Italian Society of Sciences has awarded its prize in mathematics for the year 1930 to Dr. B. Segre, docent in the University of Rome, for his researches in algebraic and infinitesimal geometry.

The Prussian Academy of Sciences has awarded a silver Leibnitz medal to Dr. Karl Scheel, professor of thermodynamics at the University of Berlin.

The Planck medal of the German Physical Society has been awarded to Professor Arnold Sommerfeld, of the University of Munich.

The Accademia dei Lincei has awarded the royal prize for astronomy jointly to Professors G. Tobetti, of the University of Florence, and V. Nobile, of the University of Naples.

The Reale Istituto Lombardo di Scienze e Lettere, Milan, has established two prizes in honor of the late Professors Carlo and Alberto Pascal, to be awarded every three years alternately for work in literature and pure mathematics. The first award, for which competition closes December 31, 1934, will be for mathematics. The prizes are restricted to Italian citizens within six years from receiving the doctorate from an Italian university.

Cambridge University has awarded its Adams prize to A. S. Besicovitch, fellow of Trinity College, for a contribution to the theory of almost periodic functions.

Professor Linus Pauling, of the California Institute of Technology, has been awarded the Langmuir prize of the American Chemical Society.

Professor Kasimir Fajans, of the department of physical chemistry at the University of Munich, has been elected an honorary member of the Royal Institution of Great Britain.

Professor Henri Lebesgue has been elected an associate of the Académie Royale de Belgique.

The following Professors have been elected corresponding members of the Reale Istituto Lombardo, U. Broggi, of Milan; A. Comessatti, of Padua; G. Fano, of Turin; A. Signorini, of Naples; and G. Vitali, of Bologna.

Professors M. Picone and A. Signorini, of the University of Naples, have been elected resident members of the Società Reale of that city.

The Pontifical Academy of Sciences has elected corresponding members Professors P. Debye of the University of Leipzig, A. Denizot of the University of Poznan, P. Straneo of the University of Genoa.

Professor B. Caldonazzo, of the University of Catania, has been transferred to the University of Florence, for the same chair of rational mechanics.

Professors L. Fantappiè, of Palermo, and A. Tonolo, of Padua, have been promoted to professorships of the calculus.

Professor Leon Lichtenstein, of the University of Leipzig, has been elected foreign member of the Scientific Society of Lwow (Lemberg).

Dr. Jakob Ackeret has been promoted to an associate professorship of aerodynamics at the Zurich Technical School.

C. Gallissot, maître de conférences in mathematics at the University of Lille, has been promoted to a professorship.

Akitsugu Kawaguchi has been promoted to a professorship of mathematics at the Hokkaido Imperial University.

Dr. H. L. Brose has been appointed to the chair of physics at the University of Nottingham, as successor to Professor P. E. Shaw, retired.

At Cambridge University, Dr. S. Goldstein, of St. John's College, has been appointed University lecturer in mathematics.

Professor G. H. Hardy, Savilian professor of geometry at the University of Oxford, has been elected to the Sadlerian professorship of pure mathematics at Cambridge University, as successor to Professor E. W. Hobson, retired.

Sir Horace Lamb, of Trinity College, Cambridge, has been appointed to the Rayleigh lectureship in mathematics.

At Cambridge University, Mr. J. M. Whittaker, of Pembroke College, has been appointed University lecturer in mathematics.

Mr. E. C. Titchmarsh has been elected to the Savilian professorship of geometry at the University of Oxford, as successor to Professor G. H. Hardy.

Princeton University announces that it has now made provision for work towards advanced degrees in mathematical physics. During the fall term of 1931–32, graduate courses in this field will be offered by Professors E. P. Adams, J. von Neumann, E. P. Wigner, E. U. Condon, and H. P. Robertson. Dr. P. A. M. Dirac, of Cambridge, will lecture on the relativistic formulation of quantum mechanics during the first part of this term.

Dr. Clifford Bell has been promoted to an assistant professorship of mathematics at the University of California at Los Angeles.

Professor B. A. Bernstein, of the University of California, has been granted leave of absence from July to December to complete papers on the foundation of mathematics and a book on the algebra of logic.

Associate Professor Thomas Buck, of the University of California, Berkeley, has been promoted to a professorship of mathematics.

Dr. C. M. Cleveland has been appointed adjunct professor of applied mathematics at the University of Texas.

Dr. D. A. Flanders, of New York University, has been promoted to an assistant professorship.

Assistant Professor Raymond Garver, of the University of California at Los Angeles, has been promoted to an associate professorship of mathematics.

Professor Temple R. Hollcroft, of Wells College, will spend his sabbatical year, 1931–32, in travel and study abroad. Mrs. Hollcroft and Temple Rice, Jr. will accompany him.

Dr. N. H. McCoy has been appointed to an assistant professorship at Smith College.

Dr. Gordon Pall has been appointed lecturer in mathematics at McGill University.

Dr. John H. Roberts, of the University of Texas, has been appointed to an assistant professorship of mathematics at Duke University.

Associate Professor C. N. Reynolds, of the University of West Virginia, has been promoted to a professorship of mathematics.

Dr. Saul Pollock, of the University of California, has been appointed to an assistant professorship of mathematics at the Indiana State Teachers College at Terre Haute.

Associate Professor Bird M. Turner, of the University of West Virginia, has been promoted to a professorship of mathematics.

Dr. E. Kathryn Wyant has been appointed professor of mathematics at the Northeastern State Teachers College, Tahlequah, Oklahoma.

The following appointments to instructorships in mathematics are announced:
Johns Hopkins University, Mr. G. R. Trott;
University of Kansas, Miss Eula Johnson, Miss Winora Venard;
Lehigh University, Mr. S. S. Cairns;
Princeton University, Mr. Jack Levine;
Temple University, Mr. J. A. Clark.

Professor Cesare Burali-Forti, of the Reale Accademia di Artiglieria e Genio, died January 21, 1931, at the age of sixty-nine.

Eugène Cosserat, formerly professor of mathematics at the University of Toulouse, and director of the observatory there, died in May, 1931, at the age of sixty-five.

Dr. Peter Pinkerton, rector of the High School of Glasgow, and for several years editor of the Mathematical Notes of the Edinburgh Mathematical Society, died November 22, 1930, at the age of sixty.

Professor Gustavo Sannia, of the University of Naples, died December 21, 1930, at the age of fifty-five.

Professor Emil Warburg, formerly president of the Federal Physical-Technical Institute, Berlin, died August 1, 1931, at the age of eighty-five.

Professor G. H. Cresse, of the University of Arizona, died May 3, 1931. Professor Cresse had been a member of the American Mathematical Society since 1912.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

278. Professor L. L. Dines and Dr. N. H. McCoy (National Research Fellow): *On linear inequalities.*

In an address which appeared in this Bulletin (vol. 36, p. 393) one of the present authors discussed linear inequalities and some related properties of functions. Subsequently several contributions to this field have been made, and also important earlier contributions have become known to the present authors. The object of this paper is to discuss these various contributions with some attempt to correlate and amplify them, and to supply a more complete bibliography of the subject than has yet appeared. (Received August 8, 1931.)

279. Professor C. R. Adams: *Transformations of double sequences, with application to Cesàro summability of double series.*

Let $\|a_{mnkl}\|$ $(m, n, k, l=1, 2, 3, \cdots; a_{mnkl}=0$ for $k>m$ or $l>n$ or both) be a matrix defining a transformation A of double sequences. Necessary and sufficient conditions that A be regular for the class of bounded sequences are known (Robison, Transactions of this Society, 1926); we show that if A satisfies these conditions, it is regular for a much larger class of sequences. The case in which $a_{mnkl}=a_{mk}'\cdot a_{nl}''$, where $\|a_{mk}'\|$ and $\|a_{nl}''\|$ are triangular matrices defining transformations A' and A'' of simple sequences, is of special interest from the standpoint of summability of double series. We then write $A=A'\cdot A''$ and prove that if A' and A'' are both regular, A is regular for the class of double sequences of which each row is transformed by A'', and each column by A', into a bounded sequence. From this flows a consistency theorem for Cesàro summability of double series which is of greater generality than those previously given. Extensions of the main result to the case in which A' and A'' are defined by square matrices, to the case in which A' and A'' are merely convergence-preserving, to general consistency questions as discussed by Hurwitz and Silverman (Transactions of this Society, 1917), and to sequences of multiplicity $n>2$, are immediate. (Received August 4, 1931.)

280. Professor G. C. Evans: *The Cauchy integral and summable boundary functions.*

The author obtains necessary and sufficient conditions that the function $w(z)=(2\pi i)^{-1}\int[f(t)+ig(t)](t-z)^{-1}\,dt$, in the region T bounded by C, take on the boundary values $f(t)+ig(t)$ for approach in the wide sense to the boundary C, at a point set of positive measure. The functions $f(t)$, $g(t)$ are real and summable (L) on C, where C is a simple closed curve, to which the Fredholm analysis of the Dirichlet problem is applicable. A necessary and sufficient condition is that $w(z)$ vanish identically outside C. An equivalent condition is that $w(z)$ be representable in terms of a potential of a double layer for a mass distribution which is absolutely continuous on C, or that the absolute continuity of $\int|w(z)||dz|$ over curves C' neighboring C and satisfying conditions analogous to those specified by Evans and Miles (American Journal of Mathematics, vol. 53, (1931), pp. 493–516), be uniform as C' approaches C. The class of these functions are equivalent to the class represented by a Cauchy integral with the alternative boundary conditions already described by the author (Cleveland Meeting, Dec. 1930). A representation in terms of the Green's function and its conjugate is permissible. Among such curves C the representation is invariant under a conformal transformation. (Received August 10, 1931.)

281. Professors C. L. E. Moore and Philip Franklin: *Dual Pfaffians.*

A Pfaffian expresses the vanishing of a linear combination of differentials which are the components of a vector. When these are replaced by the components of an $(n-1)$-dimensional element, we have a dual configuration. Similarly we have dual expressions in m- and $(n-m)$-dimensional elements. In this paper we develop the geometry of the configurations connected with such expressions. (Received July 28, 1931.)

282. Professor C. V. Newsom: *On the behavior of entire functions in distant portions of the plane.*

This paper represents an extension and a generalization of the investigations of W. B. Ford and E. W. Barnes on the asymptotic representation of entire functions when such a function is regarded as given through its Maclaurin's development. Under rather liberal restrictions upon the general coefficient of the given series it is found that the function thus defined is asymptotic to a certain definite integral. The method of the investigation, as in the work of Ford and Barnes, involves the Cauchy integral theorem and contour integration. (Received August 5, 1931.)

283. Professor C. N. Moore: *On sets of constants which are not the Fourier constants of a function integrable* (L).

The purpose of this paper is to set up a general method of constructing a set of constants, a_n, $(n=0, 1, 2, \cdots)$, where $\lim_{n\to\infty}a_n=0$, but such that the trigonometric series $\sum a_n \cos nx$ is not the Fourier development of a function that is integrable (L). In many cases the trigonometric series can be shown to converge throughout any interval of length 2π, except at a finite number of points. In the neighborhood of one or more of these points, however, the func-

tion represented by the series becomes infinite to a sufficiently high degree to cause its integral to diverge. Simple cases of such series are the following:

$$(\cos 3x)/\log 3+(\cos 7x)/\log 7+(\cos 11x)/\log 11+\cdots,$$
$$(\cos 7x)/\log 7+(\cos 13x)/\log 13+(\cos 20x)/\log 20+\cdots.$$

These examples are of interest because the series $\sum_{n=2}^{\infty}(\cos nx)/\log n$ is known to represent a function which has a Lebesgue integral over any finite interval. (Received August 10, 1931.)

284. Professor R. E. Langer: *On the asymptotic forms of the solutions of differential equations.*

In a recent paper the author has discussed the asymptotic forms of the solutions of the differential equation (1) $d^2u/dx^2+\lambda^2\phi(x)u(x)=0$, under the hypotheses: (i) that x is real, $a\leqq x\leqq b$; and (ii) that $\phi(x)=(x-x_0)^\nu\phi_1(x)$, in which $\phi_1(x)$ is real and different from zero on (a, b), while $a<x_0<b$, and $\nu\geqq 0$. This investigation is extended in the present paper to a discussion of the forms of the solutions when the variable x is complex and varies over a region of the complex plane which includes the point x_0. The restriction that $\phi_1(x)$ be real (that is, of constant argument) is also removed. Inasmuch as the Bessel equation is expressible in the form (1), the results obtained may, in particular, be specialized to yield the asymptotic forms of the Bessel functions $J_{\pm\rho}(z)$ for all complex values of the parameter ρ and the argument z. (Received August 5, 1931.)

285. Professor R. E. Langer and Dr. M. L. Hartung: *On a class of integral equations with a finite discontinuity in the kernel.*

This paper deals with the integral equation $u(x)=\lambda\int_{-\alpha}^{\alpha}K(x,\xi)u(\xi)d\xi$ whose kernel $K(x, \xi)$ has a finite non-vanishing discontinuity along the line $\xi=-x$. In a sequence of papers published individually and jointly in the Transactions of this Society between 1926 and 1928, Langer and Tamarkin have developed a theory of this equation based upon a discontinuity in the kernel or its derivatives when $\xi=x$. The extent to which the specialized location of this discontinuity governs the development of the theory has hitherto remained open. It is proved in this paper under hypotheses of a more or less general nature that there exist infinitely many characteristic values of the parameter λ, and asymptotic expressions for these values and for the characteristic functions are obtained. The problem of the expansion of an arbitrary function in a series of the solutions is also discussed. The method used consists of a transformation of the given equation into a functional system which is studied by an adaptation of methods from the theory of differential equations. Methods and results are analogous to those of the papers mentioned above and might tend to show that the location of the discontinuity may not be of major importance. (Received July 31, 1931.)

286. Professor Norman Miller: *Some extensions of the Schwarz-Stieltjes theorem of mean value and some consequences deduced therefrom.*

Among several generalizations of the mean value theorem of the differential calculus one was given by Schwarz and proved in a simple manner by Stieltjes. Using Stieltjes' method of proof some further generalizations are here stated together with particular cases of which Lagrange's interpolation formula is one. The main results of the paper flow from these theorems by limiting processes. These culminate in a generalization of Taylor's theorem with the remainder involving an arbitrary number of functions and an arbitrary number of points. (Received July 31, 1931.)

287. Professor L. M. Graves: *On the condition of Weierstrass in the problem of Lagrange.*

It is usual in the calculus of variations first to derive the differential equations which a minimizing arc must satisfy, and then to derive the Weierstrass condition by a separate construction. In the case of the isoperimetric or the Lagrange problem, this construction requires the assumption that sub-arcs of the minimizing arc are normal. The author gives a new method of proof which depends on introducing strong and weak variations simultaneously. The new proof is shorter, and it is necessary to assume only that the minimizing arc as a whole is normal. The method is still applicable when the side conditions involve integral equations. For the isoperimetric problem the method yields also a new condition in the abnormal case, provided the arc is an extremal for the integral to be minimized as well as for the integral to be kept constant. (Received July 27, 1931.)

288. Dr. W. T. Reid: *A boundary value problem associated with the calculus of variations.*

In this paper is treated the boundary value problem that arises when one seeks to minimize $I_2(\eta)=2H(\eta_1(x_1), \cdots, \eta_n(x_1), \eta_1(x_2), \cdots, \eta_n(x_2))+\int_{x_1}^{x_2}2\omega(x, \eta_1, \cdots, \eta_n, \eta_1', \cdots, \eta_n')dx$, where H and ω are quadratic in their arguments, in the class of arcs η which satisfy certain linear differential equations of the first order and certain linear end conditions, and which give a fixed value to the integral $\int_{x_1}^{x_2}\eta_i(x)K_{ij}(x)\eta_j(x)dx$. By considering the minimum of $I_2(\eta)$ in successive classes of admissible arcs the existence of infinitely many characteristic numbers for the boundary value problem is established when the matrix $\|K_{ij}\|$ satisfies rather general conditions. There are also given some expansion theorems in terms of the characteristic solutions. A boundary value problem of the type considered in this paper, and in which $\|K_{ij}\|$ reduces to the unit matrix, has recently been treated by Morse in the study of sufficient conditions for the problems of Lagrange and Mayer with variable end points; for such problems the second variation has the form $I_2(\eta)$ when the non-tangency condition is satisfied (M. Morse, American Journal of Mathematics, vol. 53 (1931), pp. 517–546). (Received August 7, 1931.)

289. Professor C. R. Adams: *Note on multiple Dirichlet and multiple factorial series.*

A recent paper (Annals of Mathematics, 1931, pp. 67–82) in which the writer developed some of the fundamental properties of multiple factorial series left

open the question of the validity for multiple series of the natural analogue of the well known theorem concerning simple series: A factorial series in x and the associated Dirichlet series both converge or both diverge at each point $x(\neq 0, -1, -2, \cdots)$ of the complex plane. It turns out that the natural analog is not valid, and the purpose of this note is to investigate completely the relation between the regions of convergence of the two associated multiple series. It is first shown that the region of convergence of the double Dirichlet series $\sum_{i,j=1}^{\infty} c_{ij} e^{-\lambda_i x - \mu_j y}$, like that of the double factorial series, is a pair of related right half-planes. The interiors of the regions of convergence of the two associated series are then identified, after which it is proved by example that one series may converge and the other diverge at a boundary point. The extension to series of multiplicity $n>2$ is immediate. (Received August 4, 1931.)

290. Professor J. A. Shohat: *On Stieltjes continued fractions.*

Let $\psi(x)$ be monotonic and non-decreasing on (a, b) $(b \geqq a > 0, b$ finite or infinite). Let us consider the Stieltjes continued fraction for $\int_a^b dy(y)/(x-y) = 1/(e_1 x + 1/(e_2 + 1/(e_3 x \cdots)\cdots)\cdots)$, $(e_i = \text{const.})$. The object of the present paper is to investigate the relation of the denominators of its odd convergents to those of its even convergents, the latter being well known as the orthogonal Tchebycheff polynomials $\phi_n(x)$ for which $\int_a^b \phi_m(x) p_n(x) dx(x) = 0$, $(m \neq n)$. The relation in question is found to be very simple and leads to some interesting applications in mechanical quadratures. The paper will appear in full in the American Journal of Mathematics. (Received July 15, 1931.)

291. Professor J. A. Shohat: *On some applications of Parseval's formula.*

Let $\{f_n(x)\}$, $(n = 0, 1, 2, \cdots)$, be a system of orthogonal normal functions on (a, b), for which Parseval's formula holds true: $\int_a^b f^2(x) dx = \sum_{n=0}^{\infty} (\int_a^b f(x) f_n(x) dx)^2$. By specifying $f(x)$ some interesting results are obtained. In particular, taking for $\{f_n(x)\}$ the polynomials of Legendre, Laguerre, etc. yields the sum of certain numerical series, as well as the solution of certain types of problems in the calculus of variations. (Received July 15, 1931.)

292. Professor Harry Bateman: *The k-function, a particular case of the confluent hypergeometric function.*

The k-functions satisfy the differential equation $xy'' = (x-n)y$ and form a special class of confluent hypergeometric functions mentioned by Whittaker as needing investigation. By using a definite integral to define the k-function, its properties are easily developed and some definite integrals evaluated. An interpolation formula provides an alternative method of development. (Received June 13, 1931.)

293. Professor J. D. Tamarkin: *On a theorem of S. Bernstein-Widder.*

An important theorem giving necessary and sufficient conditions for the representability of a function by means of a Laplace-Stieltjes integral was discovered recently by S. Bernstein and, subsequently but independently, by D.

V. Widder. Widder's proof is essentially simpler than, and is based on considerations entirely different from Bernstein's. In the present note, which was suggested by the reading of Widder's paper which will appear in the Transactions of this Society, it is shown that Widder's proof can be made still simpler and shorter. (Received August 5, 1931.)

294. Professor G. C. Evans: *Complexes and dimension theory.*

The author constructs two systems of postulates for a theory of dimension, in each case independent among themselves. The dimension is defined for every set, but is not always the same as that of Menger, although the dimension of a continuous arc is conserved as unity. The postulates are, of course, based on those of Menger, but isotopic transformations are used instead of homeomorphic, and complexes instead of compact sets. In one system a denumerable set has positive dimension and the set of all rational points in R_3 has dimension 3; in the other system, dimension zero. In neither system is a set of positive measure (L) in R_3 necessarily of dimension 3. (Received August 10, 1931.)

295. Professor W. E. Roth: *On the equation $P(A, X)=0$ in matrices.*

The equation

$$P(A, X) \equiv \sum_{k=0}^{p} F_k(A) X^{p-k} = 0,$$

where $F_k(\lambda)$, $k=0, 1, \cdots, p$, are polynomials or analytic functions in λ and where A and X are square matrices, is here solved for X. Inequalities limiting the number and degree of the elementary divisors of the characteristic determinant, $|X-\mu I|$, of a solution are established. They depend only upon the character of the polynomial $P(\lambda, \mu)$ and upon the number and degree of the elementary divisors of $|A-\lambda I|$. Solutions of the given equation are considered under the hypotheses: namely, (a) that X be a unilateral solution, (b) that X be a bilateral solution, and (c) that X be commutative with A. The effect of these successive restrictions upon X is clarified and the difficulties which must be overcome to establish necessary and sufficient conditions for the existence of solutions under any of these hypotheses are exposed. (Received August 5, 1931.)

296. Professor C. G. Latimer: *A correspondence between classes of matrices and classes of ideals.*

All matrices considered in this paper are square matrices of the same order n. Unless otherwise stated, the matrices have rational integral elements only. The matrices A, A_1 will be said to belong to the same class if there is a matrix Z, $|Z|=\pm 1$, such that $A_1=ZAZ^{-1}$. Let $\phi(x)=x^n+c_1x^{n-1}+\cdots+c_n$ be irreducible, the c's being rational integers. If $\phi(A)=0$, then $\phi(A_1)=0$. We show that there is a one-to-one correspondence between the classes of matric solutions of $\phi(x)=0$ and the classes of ideals in the algebraic field defined by a root of the same equation. This result is extended to the case where the c's are integers in an algebraic field f, of class number unity. This paper will appear in vol. 33 (1932) of the Annals of Mathematics. (Received August 10, 1931.)

297. Professor H. S. Vandiver: *On the method of infinite descent and Fermat's last theorem for regular prime exponents.*

In this paper the author applies the method of infinite descent to obtain a proof of the Last Theorem for regular prime exponents in the first case of the theorem, that is, when in the relation $x^\chi+y^\chi+z^\chi=0$, xyz is prime to the odd prime χ. Heretofore the method of infinite descent has been used only for the proof of the theorem when one of the integers x, y, z is divisible by χ. (Received August 6, 1931.)

298. Professor W. L. Ayres: *Note on a property of continuous arcs.*

In a recent paper (Proceedings Cambridge Philosophical Society, vol. 26, p. 31) Mr. S. Verblunsky proves the following result. Let C be a continuous arc in the plane joining the points p and q and suppose that C does not cross the line pq. Then given any positive number $h \leqq \rho(p, q)$ there exist two points b and c of C such that $\rho(b, c)=h$ and the line bc is parallel to the line pq. His proof involves properties of the function $\log(z-a)$ in the complex plane and the fact that an arc may be approximated by a sequence of polygonal lines of a certain type. In this note a simple proof is given following directly from the order properties of points on an arc. Also the theorem is generalized by replacing the arc by any bounded continuum and replacing the condition that C does not cross the line pq by the hypothesis that no point of the segment (p, q) is a limit point of points of C lying on one side of the line pq. (Received July 24, 1931.)

299. Dr. E. W. Miller: *On a kind of homogeneity in continuous curves.*

A continuum C will be called homogeneous with respect to points of the same (Urysohn-Menger) order if for every two points p and q of C which are of the same order there is a $(1-1)$ bicontinuous transformation T of C into itself such that $T(p)=q$. In this paper conditions are given which are necessary and sufficient that an acyclic continuous curve have the above property. (Received August 7, 1931.)

300. Mr. L. E. Bush: *Algebras without moduli which are equivalent to the algebras of their first and their second matrices.*

The purpose of this investigation is to answer the question as to the existence of linear associative algebras over an infinite field which, although not possessing moduli, are equivalent to the algebras of both their first and their second matrices. The index of an element of an algebra with respect to the algebra is defined and some of its properties are found. By the use of these indices and the decomposition of the algebra with respect to a principal idempotent element certain necessary conditions are derived for an algebra to be of the kind sought. With the aid of these an example of such an algebra is constructed. For a certain special type of algebras both necessary and sufficient conditions are found. (Received July 26, 1931.)

301. Professor H. Hasse: *Theory of cyclic algebras over an algebraic field.*

Cyclic (Dickson) algebras over an algebraic field Ω of finite degree and defined by a cyclic field Z of order n and a number α in Ω are considered. The problem of characterizing such an algebra by means of a system of total invariants is completely solved. These results are applied to prove that the exponent of any cyclic algebra is its index, that the direct product of any two cyclic algebras is a cyclic algebra, and that the Dickson-Wedderburn sufficient condition that a cyclic algebra be a division algebra is also necessary. Finally it is shown that if an algebra is cyclically representable it is also cyclic. (Received August 3, 1931.)

302. Professor Raymond Garver: *On rational transformations.*

When a rational (non-integral) transformation is applied to the roots of an algebraic equation the setting up of the transformed equation is usually a rather cumbersome matter. This paper discusses devices which are useful when the denominator of the transformation is of a certain type. An important transformation on the Brioschi quintic is given special study. (Received August 10, 1931.)

303. Professor H. S. Vandiver: *On the second factor of the class number of a cyclotomic field.*

In this paper the author proves the following theorem. A necessary and sufficient condition that the second factor of the class number of the field defined by $\zeta = e^{2i\pi/l}$ be divisible by l is that at least one of the units E_i, $(i = 1, 2, \cdots, l_1;$ $l_1 = (l-3)/2)$, be the lth power of a unit in $k(\zeta)$. Here $E_i = \epsilon^R$; $R = r^{l^2} + sr^{l^2-2i} + s^2r^{l^2-4i} + \cdots + s^{l_1}r^{l^2-2il_1}$; $\epsilon = \left\{\frac{1}{l}(1-\zeta^r)(1-\zeta^{-r})/((1-\zeta)(1-\zeta^{-1}))\right\}^{1/2}$; r is a primitive root of l and we are employing the Kronecker-Hilbert notation of symbolic powers, the substitution s denoting (ζ/ζ^r). An abstract of this paper appeared in the Proceedings of the National Academy of Sciences (vol. 16 (1930), pp. 743–749). (Received August 6, 1931.)

304. Professor H. S. Vandiver: *On the norm-residue symbol in the theory of cyclotomic fields.*

The norm-residue symbol (Hilbert, Sitzungsberichte, D. M. V., 1894, p. 413) has been treated by a number of writers, but so far, apparently, its value has always been represented by a summation. In this note the author gives a method for carrying out these summations. An abstract of this article appeared in the Proceedings of the National Academy of Sciences, vol. 16 (1930), pp. 740–743. (Received August 6, 1931.)

305. Professor H. S. Vandiver: *Summary of results and proofs on Fermat's Last Theorem.* (Sixth paper.)

In this article the author states a number of new results concerning the Last Theorem with proofs indicated in most cases. Theorems in articles already published are shown to yield proofs of the Last Theorem for all exponents less than 307. (Received August 6, 1931.)

306. Mr. C. H. Harry: *Concerning non-locally compact spaces.*

In his paper *The cyclic and higher connectivity of locally connected spaces* (American Journal of Mathematics, vol. 53, No. 2, April, 1931, p. 442, Prob. (10.2)) G. T. Whyburn raises the question as to whether or not, given any two points a and b of a locally connected and connected space M having no cut point, two mutually exclusive connected open sets R_1 and R_2 exist such that $a+b \subset \overline{R}_1 \cdot \overline{R}_2$. The present paper contains an example showing that this is not always possible. Also included in the paper is the following theorem: If M is any connected and locally connected space having no cut point and a any point of M such that a together with no other point z separates the space, then for any two other points x and y neither of which is a local end point there exists a region R, containing a, such that x and y lie together in the same component C of $M-\overline{R}$, and furthermore, no point p separates x and y in C. (Received August 5, 1931.)

307. Professor W. A. Wilson: *On cyclic numbers of one-dimensional compact sets.*

In this paper is given a modification of Alexandroff's definition of the connectivity number of a one-dimensional compact continuum. This is used to derive theorems relative to the cyclic numbers of a one-dimensional compact set and its components and the divisor of a sequence of such sets, and an addition theorem valid for sets whose cyclic and component numbers are finite. (Received July 13, 1931.)

308. Professor W. A. Wilson: *On upper semi-continuous decompositions of compact continua.*

It is shown that for a certain class of compact continua there is a unique upper semi-continuous decomposition into certain disjoint sub-continua as elements such that the original continuum is locally connected about each element and the decomposition is the finest possible such decomposition. The elements are sub-continua composed of oscillatory sets of the given continuum and the class of continua for which the theorems are valid embraces all simply connected one-dimensional compact continua as a sub-class. (Received July 13, 1931.)

309. Professor G. T. Whyburn: *A junction property of locally connected sets.*

In this paper it is shown that if M is any connected locally connected separable metric set and K is any self-compact subset of M, then M contains a subset H which is dense in K and is the image under uniformly continuous transformation of the set D of all dyadic rational numbers on an interval. As a corollary we have that if M is a subset of some complete space, then $\overline{H}$ is a compact locally connected continuum (since it is the continuous image of an interval) and of course it contains K. With the aid of our theorem we deduce the Moore-Menger theorem on the arcwise connectivity of complete locally connected sets from the well known arcwise connectivity of compact locally

connected continua. Also we prove in this paper that if A and B are relatively closed sets such that $A+B$ has k components and $A \cdot B$ has n components, then A and B themselves have finite numbers n_a and n_b respectively of components and $n_a+n_b \leqq n+k$. (Received August 10, 1931.)

310. Professor G. T. Whyburn: *Regular chains and their application to proofs in point set theory.*

For any $\epsilon>0$, a connected set C which is the sum of a finite number of ϵ-continua no two having infinitely many points in common will be called a regular ϵ-chain. The existence of locally connected regular ϵ-chains joining any two given points in very general locally connected sets is readily established. Also it follows at once that the product of any monotone decreasing sequence of regular ϵ-chains is regular if $\epsilon \to 0$ and thus, when connected, is locally connected. Advantage is taken of these facts to obtain very simple and easily followed proofs for the arcwise connectivity of locally connected continua and for the more general Moore-Menger theorem in complete spaces. (Received August 10, 1931.)

311. Professor A. J. Maria: *Examples of harmonic functions.*

The author adapts an entire function of Malmquist to show the existence of a not identically vanishing function, harmonic in the unit circle, which approaches, in the wide sense, zero at each boundary point. An example is also given for the sphere. It is further shown, by means of an example, that a function, analytic in the unit circle, which takes on, in the wide sense, summable boundary values at each point, is not necessarily given by a Cauchy integral. (Received August 10, 1931.)

312. Dr. R. S. Burington: *An invariantive classification of plane cubic curves under the affine group.*

The first classification of cubic curves given by Newton is open to objection on a number of grounds: (1) degenerate cases are not treated; (2) the classification is not invariantive and harbors certain ambiguities; (3) the rules for distinguishing species are unsatisfactory. In this paper we use as a basis for an invariantive classification of plane cubic curves under affine transformations a certain conic covariant C_2. (See Annals of Mathematics, December, 1928, pp. 52–60.) The cubic is reduced by means of the affine group to 42 canonical forms in 8 cases which are further divided into 110 species. Case I is divided according to the types of diameters which the cubic has for chords parallel to each asymptote A_i. Each A_i leads to a certain quartic covariant $E_i(x)=0$ and associated discriminant matrix M_i whose rank and signature are used extensively to separate cubics into species. The other cases are treated similarly. An extended abstract of this paper will appear in "Abstracts of Dissertations" (Ph.D), The Ohio State University Press, June 1931, No. 6. (Received July 29, 1931.)

313. Professor H. V. Craig: *On a covariant differentiation process.*

This paper deals with a covariant differentiation process for functions which involve, in addition to the coordinate variables, derivatives of these variables with respect to a parameter. The process is based on a connection due to J. H. Taylor. (Received July 29, 1931.)

314. Professor Lincoln LaPaz: *Characteristic properties of the euclidean length integral.*

In a paper which will soon appear in the Proceedings of the National Academy of Sciences, the writer has proved that the problem of minimizing the euclidean length integral is the only one for which transversality is orthogonality and the extremals are straight lines. In the present paper an independent proof of this result is given by a method of importance in the study of those inverse problems of the calculus of variations in which the extremal system is prescribed before any specification is made of the transversality relation as, for example, in the case of the Hamel geometries. (Received August 7, 1931.)

315. Professor C. J. Coe: *Vector treatment of the displacements of a rigid body.*

This paper presents what are believed to be new and very simple vector proofs of the basic theorems concerning the finite displacements of a rigid body. The discussion presupposes only an understanding of the elements of vector arithmetic and nowhere involves explicitly such higher concepts as tensor, screw, quaternion, etc. The treatment rests on a vector form of Rodrigues' formula which connects the initial and terminal positions of a point of a rigid body when the body is subjected to a displacement leaving one point fixed. This is first employed to furnish a simple vector proof of Euler's theorem on the displacements of a rigid body with one fixed point. Rodrigues' formula next yields a very simple demonstration of the formula for composition of finite rotations of a body with one fixed point. Chasles' theorem that the most general displacement of a rigid body may be effected by a translation along an axis followed by a rotation about that axis is next proved, followed by a derivation of the formula for the composition of such twists. All these are then extended to the corresponding theorems on infinitesimal displacements. (Received August 1, 1931.)

316. Professor E. V. Huntington: *A simplified theory of the figure of the earth.* (Preliminary communication.)

This paper shows that a rigorous formula for the figure of the earth (the theoretical form which the rotating earth would have if its surface were smooth) can be developed without any serious assumptions in regard to the distribution of matter inside the earth, and without using any mathematical theory more advanced than the calculus. (The theory of potential, and Clairaut's theorem are not required.) By using the concept of the (variable) *center of attraction* of the earth corresponding to any point on the surface, the parametric equations of the generating curve of the earth-revolute are found to be $x=a\cos\theta$, $y=b\sin\theta(1+fk\cos^2\theta)$, where a=the equatorial radius, b=the polar radius,

$f=(a-b)/a$, and $k=$a new constant, which may be called the redundance of the earth, given by $fk/4=(r_q-R_q)/a$, where r_q is the quadrantal radius vector of the earth-curve, and R_q is the quadrantal radius vector of an ellipse having the same semi-axes. If the earth were an ellipsoid of revolution, k would be zero. The paper gives the necessary formulas for determining the constants a, b, and r_q (or a, f, and k) in terms of measurements made on the surface of the earth, and for connecting the parameter θ with the observable latitude ϕ; and also a formula for "g" as a function of the latitude. (Received August 18, 1931.)

317. Professor E. H. Hildebrandt: *Systems of polynomials connected with the Pearson differential and difference equations.*

This paper is based on the fact that if y is a non-identically vanishing solution of the Pearson differential equation $(b_0+b_1x+b_2x^2)dy/dx=(a_0+a_1x)y$, then the expression $(b_0+b_1x+b_2x^2)^nd^ny/dx^n$ when divided by y defines a polynomial of degree at most n, and consequently a system of polynomials. The Hermite, Laguerre and Jacobi polynomials are special cases of such systems. On the basis of their definition, in terms of the differential equation above and the derivative process, a set of relations, differential and recurrence equations, can be deduced including as instances the well known relations for the special classes of polynomials mentioned. The second part of the paper discusses a similar process with respect to the difference equation $(b_0+b_1x+b_2x^2)\Delta u=(a_0+a_1x)u$ and deduces relations parallel to those of the first part of the paper. (Received August 1, 1931.)

318. Professor P. R. Rider: *On the distribution of the correlation coefficient in small samples.*

This paper studies the frequency distribution of values of the correlation coefficient computed from small random samples from various bivariate populations. All evidence seems to point to the conclusion that the type of the sampled population has very little to do with the distribution, which is essentially the same as if the samples were drawn from a normal population. (Received August 7, 1931.)

319. Professor A. H. Copeland: *The probability limit theorem.*

This paper is based on the concept of the *Kollektiv* introduced by von Mises. Each Kollektiv defines a probability distribution function. Corresponding to the operation of averaging a group of measurements, there is a law for combining the distribution functions so as to obtain the distribution function for the average. There are many elegant proofs showing that the resulting distribution function approaches the normal law as the number of measurements becomes infinite. We are not, however, justified in asserting that the resulting Kollektiv defines this distribution function. As it is the Kollektiv which is of physical importance, an essential part of the reasoning has been omitted. This paper contains a discussion of the conditions under which the above assertion is justified. There is also a simple derivation of the probability limit theorem requiring no assumption of the existence of moments higher than the second. (Received August 10, 1931.)

320. Dr. G. L. Edgett: *Frequency distributions with given statistics which are not all moments.*

A frequency curve has many well known statistics. The term *statistics* is here used for what is sometimes called a statistical constant or statistical parameter. Some of these may be expressed in terms of moments and some may not. In fitting analytic curves to observed distributions usually a curve is found for which the moments of the observed data are the same as the moments of the analytic curve. In this paper analytic curves are found having statistics in common with the observed data which are not all moments or statistics reducible to moments. Three different methods of finding such analytic curves are considered: first, by means of a differential equation; second, by means of a transformation of the normal curve of error; and, third, by means of a converging series. By the first method five analytic curves were found having given statistics in common with the raw data; by the second method, four analytic curves were found; by the third method, two analytic curves were found. The statistics used were some combination of the mean, standard deviation, mode, median, first quartile, and third quartile. (Received August 8, 1931.)

321. Dr. C. C. Craig (National Research Fellow): *On the Tchebycheff inequality of Bernstein.*

Tchebycheff's inequality in mathematical statistics gives for the probability that the deviation x of a quantity from its mean shall not exceed in absolute value a given multiple of its standard deviation a lower limit which is often unsatisfactory. Among others, S. Bernstein has imposed conditions on the distribution of x which lead to an improved limit, but his work, being published in Russian, has escaped attention outside of Russia. He showed that if x is the sum of the independent variables, $x_1, x_2, \cdots, x_n$, and if the mathematical expectation of $|x_i|^k$ is $\leqq (\sigma_{x_i}^2/2)k!\, h^{k-2}$ for $k \geqq 2$, $i = 1, 2, \cdots, n$, in which σ_{x_i} is the standard deviation of x_i and h is arbitrary, then the probability P of $|x| < t\sigma$ satisfies $P \geqq 1 - ze^{\mu}$, where $\mu = -t^2/(2+2ht/\sigma)$. The present paper briefly reproduces Bernstein's development and then shows that the same inequality can be reached under considerably less severe conditions; namely, that either the moments or the semi-invariants of x obey the above condition on the absolute moments and that the tails of the frequency law for x are well-behaved. (Received August 10, 1931.)

322. Professor Raymond Garver: *Two notes on the mathematics of finance.*

This paper consists of two unrelated parts. The first considers the application of addition-subtraction logarithms to a number of computations, particularly that of interest rates in annuity problems. The second gives a formula for evaluating a certain type of property which is a generalization of the well known formula for the evaluation of wasting assets. (Received August 10, 1931.)

323. Professor E. T. Bell: *Arithmetical composition and inversion of functions over classes.*

The compositions referred to depend upon two operations satisfying the associative law. The range of the independent variables is general, or abstract. This introduces a new type of number which is a matrix of any number of rows and columns, the elements in a given row being compounded according to a specific composition of the above kind, and the compositions for the several rows being not necessarily identical. Such numbers include rational integers as a very special case. When the number of decompositions of a fixed element is finite, the composition is said to be arithmetical. This corresponds to unique factorization in arithmetic. Given a function whose arguments are abstract, except that decomposition is arithmetical as above, it is required to decompose the function into a composite of similar functions. This is the problem of inversion. A complete solution is given, which includes all of the numerous arithmetical inversions in the literature, and which provides the means for devising an infinity of new inversion processes. The paper will appear in the Transactions of this Society. (Received August 13, 1931.)

324. Professor Tibor Rado: *On the uniqueness of the solution of the problem of Plateau.*

The object of this paper is to generalize a uniqueness theorem given previously by the author (Proceedings of the National Academy of Sciences, vol. 16 (1930), p. 247), and to point out new examples of cases where the solution is not unique. (Received August 21, 1931.)

325. Professor G. T. Whyburn: *Concerning S-regions in locally connected continua.*

Let M denote a locally connected compact metric continuum and call any region (=connected open set) in M which has property S (that is, which is the sum of a finite number of arbitrarily small connected sets) an S-region. The following sequence of theorems is established. (1) If p is any point of M and R_ϵ is the set of all points x of M which can be joined to p by a simple chain of connected sets $L_1, L_2, \cdots, L_n$, where $\delta(L_i) < \epsilon - \sum_1^i \delta(L_j)$, then R_ϵ is an S-region; furthermore if $\sigma < \epsilon$, $\overline{R_\sigma} \subset R_\epsilon$. (2) Each point of M is contained in an arbitrarily small S-region whose exterior has only a finite number of components. (3) Any region in M with a totally disconnected boundary is an S-region. (4) Every region in M is homeomorphic with an S-region in some M-space. (5) If M has no local separating point, then every point x of M is contained in an arbitrarily small S-region R such that $\overline{R}$ has no local separating point. (Received August 25, 1931.)

326. Dr. Eberhard Hopf: *Theory of measure and invariant integrals.*

This paper is concerned with the intrinsic characterization of transformations which possess an invariant integral. Let M be a manifold and let T be a $1-1$ transformation of M into itself, which carries measurable point sets and sets of zero measure into point sets of the same kind, respectively. An invariant integral may be associated with such a transformation $\int_a f(P)dm = m^*(a)$, $m^*(a) = m^*(T(a))$, where $f(P)$ is measurable and positive almost everywhere,

while a is any measurable set of points. Transformations of this kind are well known to play an important rôle in dynamics. Two measurable point sets a and a' are called images by division of each other if a, as well as a', can be subdivided into denumerably many parts, so that each part of a' is an image of a part of a under T or a proper power of T, and conversely. The paper contains a proof of the following theorem. A necessary and sufficient condition for the existence of a finite invariant integral over M is that $m(M-M')=0$, for each image by division M' of M. (Received August 27, 1931.)

327. Professor G. C. Evans: *Poisson's equation with relation to sub-harmonic functions.*

The author continues the study of Poisson's equation in the form of Gauss's law for an arbitrary distribution of finite mass in a region T bounded by a simple closed curve C, the latter having a continuously turning tangent such that the angle between neighboring tangents is dominated by a constant times the element of arc. He shows that $U_1=\int_T \log \frac{1}{r}d\phi(e)$, where $\phi(e)$ is the mass function, takes on summable boundary values for approach along normals to the boundary C at almost all points, and that the absolute continuity of $\int U_1\,ds$ along arcs of curves in T, near C, is uniform over a sequence of curves which tends to C. Thus he is able to specify, as necessary and sufficient conditions, classes of functions in which the ordinary and generalized discontinuous Dirichlet problems are uniquely solvable. The generalized Newmann problem, in which the total flux is given on arcs of the boundary, is also uniquely solvable; but the problem is apparently too general to allow a determination in terms of the normal derivatives themselves. The methods apply to an arbitrary number of dimensions, and depend essentially on the fact that U_1 is the difference of two sub-harmonic functions.

328. Dr. Selby Robinson: *Covering theorems in general topology.*

Generalizations of the Borel theorem to abstract spaces are usually in the form of necessary and sufficient conditions for the reducibility of a proper covering of a set E. Such conditions involve either nuclearity or the closure of decreasing sequences of sets. In the present paper a study is made in general topological spaces of the reducibility of a covering to another of the same or a different kind, and of the reducibility of a covering of the space to a covering of a given set. Thus a number of new equivalences are found. The results are applied in connection with the generalization and extension of certain equivalences due to Sierpinski. A necessary and sufficient condition for the absolute closure of a space is extended to general topological spaces.

329. Professor A. A. Albert: *Normal division algebras of order sixteen over an algebraic field.*

The most important algebras for their applications are normal division algebras over algebraic fields $R(x)$ where x is a root of an equation with rational coefficients and irreducible in R, the field of all rational numbers. All normal division algebras of order four and nine have been shown by L. E.

Dickson and J. H. M. Wedderburn respectively to be cyclic (Dickson) algebras. In the present paper the author completes the theory of the next case by showing that all normal division algebras of order sixteen over $R(x)$ are cyclic (Dickson) algebras. (Received July 27, 1931.)

330. Professor J. A. Shohat: *On interpolation.*

In the Göttinger Nachrichten for 1916 (*Ueber Interpolation*, pp. 66–91) Fejér has given an interpolation formula (different from Lagrange's) which possesses many interesting convergence properties, if the points used therein are zeros of Legendre or trigonometric polynomials, whose degree increases indefinitely. The present paper shows that the aforesaid convergence properties hold if the points used are zeros of *any* of the classical orthogonal polynomials (of Jacobi, Laguerre, Hermite). The analysis, based upon the theory of mechanical quadratures, yields some interesting convergence properties also for the ordinary Lagrange interpolation formula. (Received September 9, 1931.)

331. Mr. Alexander Tartler: *On a certain new class of orthogonal polynomials.*

It is known that in the theory of orthogonal Tchebycheff polynomials, defined by $\int_a^b \phi_m(x)\phi_n(x)d\psi(x) = \delta_{mn}$, the characteristic function $\psi(x)$ is assumed to be non-decreasing in the interval considered. Recently, Professor J. A. Shohat announced (Comptes Rendus, vol. 191, p. 474) without proof certain theorems and formulas for the case when $\psi(x)$ is no longer monotonic. The purpose of the present paper is to demonstrate his results and to carry the investigation further, particularly when $d\psi_1(x) \equiv (x-\alpha)d\psi(x)$, $a<\alpha<b$. Necessary and sufficient conditions are given for the existence of a uniquely determined system of orthogonal and normal polynomials $\{\phi_n(x)\}$, $n=0, 1, 2, \cdots$. The distribution of the zeros of $\{\phi_n(x)\}$ and their dependence on α is investigated. Application to mechanical quadratures and certain asymptotic properties are given. Finally the case when $d\psi_1(x) \equiv (x-\alpha)(x-\beta)d\psi(x)$, $a<\alpha$, $\beta<b$, is considered. (Received September 9, 1931.)

332. Professor H. S. White: *Seven lines in the real projective plane.*

When straight lines are continuous at infinity, seven lines with no three on a point divide the real plane into twenty-two convex and void polygons. If none of these are hexagons or heptagons, these are of only three types or species, all actual; passage from one type to another, under the group of all rotations of the seven lines, being possible only through a coincidence of two lines or of three points. Proof is deductive for four, five, and six lines, partly observational for a seventh. The object is partly to initiate a study of Aronhold sets of bitangents to a quartic. (Received September 9, 1931.)

333. Mr. C. Winston: *On mechanical quadratures of Gauss' type.*

This paper consists of three parts. The first deals with a study of the coefficients H_i in the mechanical quadrature formula

$$\int_a^b f(x)p(x)dx = \sum_{i=1}^{n} H_i f(x_i),\ H_i = \int_a^b \{\phi_n(x)p(x)/[(x - x_i)\phi_n'(x_i)]\}dx,$$

$\phi_n(x_i)=0$, where the $\phi_n(x)$ are orthogonal Tchebycheff polynomials corresponding to the characteristic function $p(x)$ and the interval (a, b). Using a method due to Sonine (Memoirs, Russian Academy of Sciences, vol. 69(1892)) limits and order of magnitude, with respect to n, have been obtained for the H_i for the classical orthogonal polynomials (of Legendre, Jacobi, Laguerre, and Hermite). The second part considers a mechanical quadratures formula of the above type with an arbitrary number of points x_i fixed ($x_i \leqq a$ or $\geqq b$). In the last part are found limits for the roots of the generalized Laguerre polynomials ($p(x)=e^{-x}x^{\alpha-1}$, $\alpha>0$, $(a, b)=(0, \infty)$), following a method employed by E. R. Neumann for the special case $\alpha=1$ (Jahresberichte der Deutschen Mathematiker-Vereinigung, vol. 30 (1921)). (Received September 9, 1931.)

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Memorial des Sciences Mathématiques

Edited by HENRI VILLAT

Under the patronage of la Société mathématique de France, l'Académie des Sciences de Paris, and the scientific academies of Servia, Belgium, Bucarest, Poland, Ukraine, Spain, Prague, Rome (dei Lincei), and Sweden. See this BULLETIN, vol. 31, Nos. 5-6, p. 281.

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26. B. GAMBIER, *Déformation des surfaces étudiées du point de vue infinitésimal.* 1928.
27. P. APPELL, *Le problème géométrique des déblais et remblais.* 1928.
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THE RHIND MATHEMATICAL PAPYRUS, British Museum 10057 and 10058, in two volumes. Volume I. By Arnold Buffum Chace, with the assistance of Henry Parker Manning, and with a bibliography of Egyptian mathematics by Raymond Clare Archibald. 210 pp. 1927. Royal 8vo. Volume II. By Arnold Buffum Chace, Ludlow Bull, and Henry Parker Manning, with a bibliography of Egyptian and Babylonian mathematics (supplement) by Raymond Clare Archibald, and a description of the mathematical leather roll in the British Museum, by S. R. K. Glanville. 31 photographic plates, 109 facsimile plates, 109 facing pages of text, 12 pp. of bibliography, 8 pp. 1929. Royal oblong folio. Price $20.00 the set, postage extra.

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The scientific section will be edited by the scientific faculty of the University of Pisa, under the direction of Professor Leonida Tonelli, Director of the Mathematical Institute of the University and Professor at the Scuola Normale. It aims to become a journal of international scope, in the range of its contributors as well as in the distribution of its readers. It will publish one volume a year, consisting of four parts, of about 100 pages each. The price of subscription will be 100 lire.

Subscriptions should be sent to NICOLA ZANICHELLI, BOLOGNA (ITALY). Correspondence concerning exchanges, and editorial correspondence should be addressed to Professor L. Tonelli, University of Pisa, Pisa, Italy.

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 31, 1931.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th Street, New York City, not later than October 10. In order to be printed in the Bulletin in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than September 7. By invitation of the program committee, Professor D. V. Widder will deliver an address on *Some recent developments in the theory of algebras.*

COLUMBUS, OHIO, November 27-28, 1931.

Abstracts must be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 5. All abstracts received by that date will appear in the November issue of this Bulletin. By invitation of the program committee, Professor W. C. Graustein will speak on *Parallelism of Levi-Civita in classical differential geometry,* and Professor H. M. Gehman will speak on *Homeomorphic geometry.*

CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA, November 28, 1931.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than October 31. All abstracts received by that date will appear in the November issue of this Bulletin.

NEW ORLEANS, LOUISIANA, ANNUAL MEETING, December 28-31, 1931.

Abstracts are due by November 28, 1931. Those received by November 7, 1931, will be printed in advance in this Bulletin.

The usual February Meeting in New York City will be omitted.

NEW YORK CITY, March 25-26, 1932.

Abstracts are due by March 5, 1932.

CHICAGO, April 8-9, 1932.

Abstracts are due by March 18, 1932.

STANFORD UNIVERSITY, CALIFORNIA, April 9, 1932.

Abstracts are due by March 12, 1932.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

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The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, G. W. Mullins, 501 W. 116th St., New York.

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